**Problem 1:** Find the largest rectangle that can fit inside the ellipse:
\[
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1
\]
by posing a one variable optimization problem in a standard form, and solving for the minimum analytically.

**Problem 2:** Find and classify the stationary points for:
(a) \( f(x) = x^3 + 6x^2 - 15x + 2 \), (b) \( f(x) = x^2e^x \), (c) \( f(x) = x + x^{-1} \)

**Problem 3:** Consider a beam of sectional modulus EI and length L that is pinned at both ends. A load P is applied at a distance L/3 from the left end and a load P/2 is applied at the center. Find the location and value of maximum deflection. You may use the following equation for the deflection.

![Beam Deflection Equation](image)

For \( x < a \):
\[
y = \frac{Pb}{6EI} \left[ x^3 - (L^2 - b^2)x \right]
\]
For \( x = a \):
\[
y = -\frac{Pa^2b^2}{3EI}
\]

**Problem 4:** (Refraction Law of Optics). Let p and q be two points on the plane that lie on opposite sides of a horizontal axis. Assume that the speed of light from p to the horizontal axis is \( v \), and from the horizontal axis to q is \( w \). Find the fastest path from p to q (the path that a light ray will take). Pose as a one variable optimization problem and solve analytically.

**Problem 5:** Prove the following: Suppose a function \( f(x) \) is infinitely differentiable and
\[
\frac{d^n f(x^*)}{dx^n} = 0 \text{ for } n = 1, 2, 3, \ldots, 2m - 1
\]
\[
\frac{d^{2m} f(x^*)}{dx^{2m}} > 0
\]
then \( x^* \) is a local minimum.
Problem 6: Use the above Lemma to decide whether 0 is a local minima for \( f(x) = x^n \) for \( n = 1,2,3,... \)

Problem 7: Suppose \( f(x_1) = f_1; f(x_2) = f_2; f(x_3) = f_3 \), where \( x_1 < x_2 < x_3 \), and \( f_1 > f_2 < f_3 \). Find an estimate for the local minimum \( x^* \) via parabolic interpolation. Your expression should involve the three points \( x_1, x_2, x_3 \) and the three function values \( f_1, f_2, f_3 \).

Problem 8: Write a Matlab function to find the minima of a 1-D function via the Golden Section method (you may use a ‘while loop’ instead of recursive calls). Your code should be of the form:

```matlab
function [xmin, fmin, nIterations] = fminViaGoldenSection_LastName(f, x0, x1, alpha, xtol, ftol, maxIterations)
where the function parameters are identical to the quadfit code made available to you. Further, alpha is a parameter that you should use in your testing. Ideally, alpha is the Golden ratio. You should study the performance of your code for alpha that is not ideal. Consider the function \( f(x) = 3 \sin(x) - x + 0.1x^2 + 0.1 \cos(2x) \) for testing.

Test and compare your code against fminbnd for accuracy and efficiency. Submit your results along with the rest of your solution, and email me your code.