Addendum to Surface Roughening Document

Non-Surface Heating

Most surface heating actually deposits heat as volumetric heating within a thin layer near the surface. A typical model for volumetric heating resulting from energy impinging on a surface is

$$Q'' = Ae^{-r \cdot x}$$

Where A is a constant, γ is the attenuation coefficient and x is the distance from the surface. To provide the same total heat input as a true surface heating flux q, we must enforce $A=q\gamma$. The temperature distribution resulting from volumetric heating of this type is

$$T = \frac{2q}{k\gamma} \left[ \text{erfc} \left( \frac{\eta}{2z} \right) - e^{-\eta} + e^{z^2 - \eta} \text{erfc} \left( z - \frac{\eta}{2z} \right) + e^{z^2 + \eta} \text{erfc} \left( z + \frac{\eta}{2z} \right) \right]$$

where $\eta=x\gamma$ and $z = \gamma \sqrt{\frac{t}{\kappa_D}}$, representing the ratio of the diffusion length in time t to the characteristic deposition length. The surface temperature resulting from this solution is

$$T_{surface} = \frac{q}{k\gamma} \left[ \frac{2z}{\sqrt{\pi}} - 1 + e^{-z^2} \text{erfc}(z) \right]$$

The ratio of the surface temperature from this equation to the surface temperature due to surface heating is

$$R = 1 - \frac{\sqrt{\pi}}{2z} \left[ 1 - e^{-z^2} \text{erfc}(z) \right]$$

Figure 1 provides a plot of this ratio as a function of z. The corresponding stresses would follow the same curve. From this curve one can see that the effect is less than 10% for $z>8$. The effect is less than 1% for $z>60$. This latter result was determined using the asymptotic result:
\[ R \sim 1 - \frac{\sqrt{\pi}}{2z} + \frac{1}{2z^2} \]

Figure 1: Ratio of surface temperatures due to volumetric heating and equivalent surface heating as a function of dimensionless time \( z \).

Using the surface temperature derived above, the fluence to just yield the surface is given by

\[
\Gamma_\gamma = \frac{Y(1-\nu)k\gamma t_p}{E\alpha \left( \frac{2z}{\sqrt{\pi}} - 1 + e^{-z} \text{erfc}(z) \right)}
\]

The fluence to cause yielding in aluminum is given in the following table for several values of pulse length and \( \gamma \).
Table 1: Fluence to just yield aluminum.

Results are plotted in the following two figures for aluminum and tungsten.