

Response to “Comment on ‘Paleoclassical transport in low-collisionality toroidal plasmas’ ” [Phys. Plasmas 14, 104701 (2007)]

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The Comment¹ mainly poses questions related to the validity and applicability of the *key hypothesis of the paleoclassical model* of radial electron heat transport, which is that electron guiding centers diffuse radially along with thin annuli of poloidal magnetic flux in resistive, current-carrying axisymmetric toroidal plasmas. While this key hypothesis was only motivated phenomenologically in the original toroidal paper,² a derivation of it has recently been published.³ The derivation is based on transforming the drift-kinetic equation from laboratory to poloidal magnetic flux coordinates in a situation where the poloidal flux obeys a diffusion equation.

Since the derivation in Ref. 3 was carried out in the full axisymmetric toroidal magnetic field geometry and is somewhat complicated, it is helpful to present and discuss a simpler model that illustrates the key points involved. These key points are crucial for addressing the main issues raised in the Comment.¹

Thus, consider the general procedure for solving a simple one-dimensional plasma kinetic equation, which is a first order partial differential equation (PDE) for $f(x, t)$ in the two variables x, t ,

$$\left. \frac{\partial f}{\partial t} \right|_x + \tilde{v}_x \left. \frac{\partial f}{\partial x} \right|_t = S(x, t), \quad \tilde{v}_x \equiv v_0 \cos(\omega t). \quad (1)$$

Here, \tilde{v}_x is an oscillatory speed in the x direction, which represents the x motion of an individual particle that contributes to the overall distribution f of such particles, and S is the source of f .

The formal approach for obtaining a general solution of this equation is to integrate along the mathematical characteristic curves of this PDE. These curves are defined by $dx/dt = \tilde{v}_x$ and have the simple (particle trajectory) solution $x = x_0 + (v_0/\omega)\sin \omega t$. To integrate along these characteristic curves (particle trajectories), prime the x and t variables, define $dx'/dt' = v'_x$ and use $df/dt' = \partial f/\partial t' + (dx'/dt') \partial f/\partial x'$ to obtain

$$\frac{df(x', t')}{dt'} = S(x', t'). \quad (2)$$

Integrating this over the running time t' using the characteristic curves $x' = x_0 + (v_0/\omega)\sin \omega t'$, one obtains

$$f(x, t) = f(x, 0) + \int_0^t dt' S(x', t'). \quad (3)$$

Now consider what happens when the \tilde{v}_x oscillation is not about an x position, but really about a given $\psi(x, t)$. Then, assuming ψ is a monotonic (i.e., invertible) function of x , the original equation can be transformed from x to the new radial coordinate $\psi(x, t)$,

$$\left. \frac{\partial f}{\partial t} \right|_{\psi} + \left(\left. \frac{\partial \psi}{\partial t} \right|_x + \tilde{v}_{\psi} \right) \frac{\partial f}{\partial \psi} = S[\psi(x, t), t], \quad (4)$$

in which $\tilde{v}_{\psi} \equiv \tilde{v}_x (\partial \psi / \partial x)$. The mathematical characteristic curves of this PDE are governed by

$$\frac{d\psi'}{dt'} = \tilde{v}_{\psi}(t') + \frac{\partial \psi'}{\partial t'}, \quad (5)$$

with initial condition that $\psi' = \psi(x_0, 0)$ at $t' = 0$.

Next, consider the case, as in the paleoclassical model, where on a long time scale compared to the oscillation period $2\pi/\omega$, $\psi(x, t)$ obeys a diffusion equation,

$$\frac{\partial \psi}{\partial t} = D \frac{\partial^2 \psi}{\partial x^2} - S_{\psi}. \quad (6)$$

Here, D is a diffusion coefficient, which for simplicity will be assumed to be spatially constant, and S_{ψ} is the source of ψ . Then, the equation for the mathematical characteristic curves (effective particle trajectories) becomes

$$\frac{d\psi'}{dt'} = \tilde{v}_{\psi}(t') + \left(D \frac{\partial^2 \psi'}{\partial x^2} - S_{\psi} \right), \quad (7)$$

whose solution embodies both hyperbolic (\tilde{v}_{ψ}) and parabolic ($D \partial^2 \psi / \partial x^2$) mathematical characteristics.

A multiple-time-scale solution of Eq. (7) will be sought. Supposing that $\tilde{v} \equiv v_0 \cos \omega t$ oscillates about a given ψ surface, D is small and $\psi' \approx \psi(x_0) + (x' - x_0) \partial \psi / \partial x|_{x_0}$, the lowest order equation is the same as before; $dx'/dt' = \tilde{v}'_x \Rightarrow x' = x_0 + (v_0/\omega)\sin \omega t'$. For time scales longer than $1/\omega$ one allows x_0 to have a slow time dependence [i.e., $x_0 = x_0(x, t)$], and begins with

$$x_0(x, 0) = x_{00} \delta(x/x_{00} - 1), \quad \text{initial condition.} \quad (8)$$

The initial condition represents localization of the initial characteristic curve to an arbitrary, nonzero x position x_{00} . Then, using the Taylor series expansion $\psi' \approx \psi(x_{00}) + (x' - x_{00}) \partial \psi / \partial x|_{x_{00}}$, the solubility condition to prevent secular growth in the solution of Eq. (7) on the long time scale ($t > 1/\omega$), which results from averaging Eq. (7) over the oscillatory period of $2\pi/\omega$, becomes

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$$\frac{\partial x_0(x, t')}{\partial t'} = D \frac{\partial^2 x_0(x, t')}{\partial x^2} - \frac{S_\psi}{\partial \psi / \partial x}. \quad (9)$$

This equation can also be obtained from Eq. (7) using the method of averaging.⁴ While the source S_ψ is important for the long time scale equilibrium, for short time scales the sharp localization of the mathematical characteristic curves near the delta-function initial condition about x_{00} causes $|\partial^2 x_0 / \partial x^2| \gg |S_\psi / (\partial \psi / \partial x)|$ for $t \ll x_{00}^2 / D$. Thus, for short time scales Eq. (9) reduces to

$$\frac{\partial x_0(x, t')}{\partial t'} \approx D \frac{\partial^2 x_0(x, t')}{\partial x^2}. \quad (10)$$

For short times compared to a global diffusion time $\tau \sim x_{00}^2 / D$ on which diffusion to boundaries occurs, the local, slablike solution of this diffusion equation is

$$x_0(x, t') = x_{00} \frac{e^{-(x-x_{00})^2/4Dt'}}{[4\pi(D/x_{00}^2)t']^{1/2}}, \quad (11)$$

which is valid for $1/\omega < t \ll x_{00}^2 / D$. The total mathematical characteristic curves are the sum of the oscillatory and diffusive contributions,

$$x' = (v_0/\omega) \sin \omega t' + x_{00} \frac{e^{-(x-x_{00})^2/4Dt'}}{[4\pi(D/x_{00}^2)t']^{1/2}}. \quad (12)$$

The diffusive effects in the mathematical characteristic curves (effective particle trajectories) can be incorporated into the kinetics in one of two ways: (1) Integrate the formal solution in Eq. (3) along the trajectories in Eq. (12); or (2) add a Fokker-Planck diffusion operator to the kinetic equation to represent these effects. The relevant spatial Fokker-Planck “diffusion” operator [$\langle \Delta x \rangle / \Delta t = 0$] is

$$\mathcal{D}\{f\} = \frac{1}{2} \frac{\partial^2}{\partial x^2} \left(\frac{\langle (\Delta x)^2 \rangle}{\Delta t} f \right) = D \frac{\partial^2 f}{\partial x^2}, \quad (13)$$

because

$$\frac{\langle (\Delta x)^2 \rangle}{\Delta t} \equiv \frac{\int_{-\infty}^{\infty} dx (x-x_{00})^2 x_0(x, t')}{\int_{-\infty}^{\infty} dx x_0(x, t')} = 2D, \quad (14)$$

and D was assumed to be constant in space. Thus, when the kinetic equation (1) is transformed from laboratory coordinates to a slowly diffusing ψ coordinate about which the particles rapidly oscillate, it becomes

$$\frac{\partial f}{\partial t} \Big|_{\psi} + \bar{v}_\psi \frac{\partial f}{\partial \psi} = \mathcal{D}\{f\} + S(\psi, t). \quad (15)$$

This kinetic equation adds the spatial diffusion operator $\mathcal{D}\{f\}$ to the right-hand side of the initial kinetic equation (1); it is valid for all times longer than the oscillation period (i.e., $t > 1/\omega$). Even though the Fokker-Planck coefficient $\langle (\Delta x)^2 \rangle / \Delta t$ was derived for short time scales compared to the “global” diffusion time scale x_{00}^2 / D , the $\mathcal{D}\{f\}$ operator is valid for all times longer than $1/\omega$, because the differential properties of the ψ and f diffusion processes are preserved in the Fokker-Planck operator. This modified kinetic equation is valid even in steady state where $\partial \psi / \partial t|_x = 0$, because (1) mathematically, $D(\partial^2 \psi / \partial x^2)(\partial f / \partial \psi)$ introduces parabolic

(diffusive) mathematical characteristics; or (2) physically, one is transforming to a locally diffusing ψ coordinate system relative to and about which a particle is oscillating.

The critical features of this derivation are that: (1) particles oscillate rapidly about a ψ surface, (2) ψ obeys a slowly evolving diffusion equation, and (3) further analysis of the kinetic equation is to be carried out on and relative to ψ surfaces. The details of the source S_ψ do not matter. The fact that it balances the diffusion term $D \partial^2 \psi / \partial x^2$ in steady state does not vitiate the argument presented here, as discussed explicitly at the end of the preceding paragraph.

Radial diffusion of the effective particle trajectories in steady-state is analogous to radial diffusion of particles governed by a simple density diffusion equation $\partial n / \partial t = D \partial^2 n / \partial x^2 + S_n$ with a spatially constant D . For a group of “marked particles” (subscript m) initially described by $n_m(x, 0) = N_m \delta(x - x_{00})$, the marked particle density $n_m(x, t)$ obeys a diffusion equation analogous to Eq. (10) and for short times ($t < x_{00}^2 / D$) has a solution like that in Eq. (11), $n_m(x, t') = N_m e^{-(x-x_{00})^2/4Dt'} / (4\pi Dt')^{1/2}$. Note again that the source (here S_n) is not relevant in the determination of the local diffusion properties of marked particles. In Eqs. (8)–(15), the marked particles are the ψ surfaces (or, more specifically, thin annuli of poloidal flux³) relative to and about which the particles oscillate.

Now consider application of this transformation procedure for a simple plasma kinetic equation to the derivation of the paleoclassical key hypothesis. First, note that in an axisymmetric toroidal magnetic field the canonical toroidal angular momentum of an electron is a constant of the motion on the gyro and bounce motion time scales:^{2–4} $p_\zeta = R^2 \nabla \zeta \cdot (m_e \mathbf{v} + q_e \mathbf{A}) = R^2 \nabla \zeta \cdot m_e \mathbf{v} - q_e \psi_p$, in which ψ_p is the poloidal magnetic flux. Since $R^2 \nabla \zeta \cdot m_e \mathbf{v}$ is oscillatory on the gyro and bounce time scales, the constancy of p_ζ shows that electrons in an axisymmetric toroidal plasma oscillate about ψ_p surfaces—thus satisfying the first requirement for the relevance of the preceding illustrative derivation.

For a resistive, current-carrying plasma in an axisymmetric toroidal magnetic field, the poloidal magnetic flux $\psi_p \rightarrow \psi$ is governed by the equation^{2,3}

$$\frac{\partial \psi}{\partial t} \Big|_x + \bar{u}_G \frac{\partial \psi}{\partial \rho} = D_\eta \Delta^+ \psi - S_\psi, \quad D_\eta \equiv \frac{\eta}{\mu_0}. \quad (16)$$

In terms of notation analogous to that in the Comment,¹ the “grid velocity” \bar{u}_G , magnetic diffusion term $D_\eta \Delta^+ \psi$, and source S_ψ of poloidal magnetic flux are given by

$$\bar{u}_G \partial \psi / \partial \rho = \langle \mathbf{u}_G \cdot \nabla \psi \rangle \equiv \langle \mathbf{E} \cdot \mathbf{B}_{\text{pol}} \rangle / \langle \mathbf{B} \cdot \nabla \zeta \rangle, \quad (17)$$

$$D_\eta \Delta^+ \psi \equiv \eta \langle \mathbf{J} \cdot \mathbf{B} \rangle / \langle \mathbf{B} \cdot \nabla \zeta \rangle, \quad (18)$$

and

$$S_\psi = -\bar{V}_\ell / 2\pi + \eta \langle \mathbf{J}_{\text{CD}} \cdot \mathbf{B} \rangle / \langle \mathbf{B} \cdot \nabla \zeta \rangle. \quad (19)$$

Since D_η is small, the poloidal flux ψ satisfies a slowly evolving diffusion equation on the bounce motion time scale; thus, the second requirement for applicability of the derivation in Eqs. (4)–(15) above is satisfied. The third requirement is also satisfied since all kinetic analyses of drift-kinetic and

gyrokinetic equations for tokamak plasmas (e.g., for neoclassical and microturbulence-induced flows and transport) are carried out relative to poloidal flux surfaces³—so the constancy of the canonical toroidal angular momentum follows readily from the lowest order mathematical characteristic curves of the drift-kinetic and gyrokinetic equations, which is critical for kinetic analyses of low collisionality plasmas where the collision frequency is much less than the bounce frequency and electrons circumnavigate the torus many times in a collision time.

Equation (16) is a simplified version of Eqs. (36), (65), and (67) in Ref. 2 in that the inertial electromagnetic skin depth effects have been neglected ($\delta_e \equiv c/\omega_p \rightarrow 0$), the neoclassical parallel resistivity $\eta_{\parallel}^{\text{nc}} \rightarrow \eta$, and the bootstrap current contribution to the source S_{ψ} is ignored. Also, the Comment¹ loop voltage $-\bar{V}_{\ell}/2\pi$ (>0 for an Ohmic-transformer-induced current) has been identified as the $V_{\text{loop}}^{\ell}/2\pi \equiv \partial\Psi/\partial t$ defined in the text after Eqs. (34) and (67) in Ref. 2, and the noninductive current source due to \mathbf{J}_{CD} has been identified as the $\eta_{\parallel}^{\text{nc}}\langle\mathbf{J}_S \cdot \mathbf{B}\rangle/\langle\mathbf{B} \cdot \nabla\zeta\rangle$ defined in the text after Eq. (67) in Ref. 2 and Eq. (2) in Ref. 3.

The diffusion equation in Eq. (16) is equivalent to Eq. (A3) in the Comment¹ in the “steady-state” limit where $\partial\psi/\partial t|_{\mathbf{x}}=0$. It is important to note that here $\psi(\mathbf{x},t)$ is the poloidal magnetic flux function within the plasma and that $(\partial\psi/\partial t)_{\mathbf{x}}$ in the Comment¹ is in general not the same as the $\partial\psi/\partial t|_{\mathbf{x}}$ used here,

$$\left(\frac{\partial\psi}{\partial t}\right)_{\mathbf{x}} \equiv \frac{\partial\Psi}{\partial t} + \frac{\partial\psi}{\partial t}\Big|_{\mathbf{x}} = -\frac{\bar{V}_{\ell}}{2\pi} + \frac{\partial\psi}{\partial t}\Big|_{\mathbf{x}}. \quad (20)$$

In the steady-state cases discussed in the Comment,¹ $\partial\psi/\partial t|_{\mathbf{x}}=0$ and $(\partial\psi/\partial t)_{\mathbf{x}}=-\bar{V}_{\ell}/2\pi$. However, the mathematical analysis in Eqs. (4)–(15) above shows that in transforming the kinetic equation from laboratory to ψ coordinates one must in general replace $\partial\psi/\partial t|_{\mathbf{x}}$ by its functional form as given on the right-hand side of Eq. (16) and not just set it to zero, because the $D_{\eta}\partial^2\psi/\partial x^2$ term in $\partial\psi/\partial t|_{\mathbf{x}}$ changes the mathematical characteristics of the kinetic equation from hyperbolic (i.e., Hamiltonian) to hyperbolic plus parabolic (i.e., diffusive).

From the form given in Eq. (19), it is clear that the discussion after Eq. (A3) in the Appendix of the Comment¹ concerns changes in the fraction $\alpha \equiv \langle\mathbf{J}_{\text{CD}} \cdot \mathbf{B}^0\rangle/\langle\mathbf{J}^0 \cdot \mathbf{B}^0\rangle$ of the poloidal flux source S_{ψ} induced by the parallel current sources that is caused by the noninductive current-drive source \mathbf{J}_{CD} . In the physically relevant case where the resistivity η is nearly constant, as α increases the inductive loop voltage decreases by a factor of $1-\alpha$ and the source of poloidal flux S_{ψ} is unchanged; hence, in a constant resistivity steady state the total current driven (or source of poloidal flux) in the plasma is unchanged as the parameter α is varied. This is because the value of α does not change the parallel current in the plasma. Hence, from Eq. (18) it does not change the amount of poloidal magnetic flux diffusion occurring in the plasma. Varying the resistivity changes the discussion some but not the key point that it is $\eta\langle\mathbf{J} \cdot \mathbf{B}\rangle \propto D_{\eta}\Delta^+\psi$ (not V_{ℓ} as is suggested in the Appendix of the

Comment¹) that is critical for the radial diffusion of guiding centers with the magnetic diffusion coefficient D_{η} in the paleoclassical model.

The Comment¹ poses three puzzles for a noninductive steady-state (NISS) tokamak plasma in which the \mathbf{E} and \mathbf{B} fields are “static” and the parallel current is totally driven by a noninductive source (i.e., $\alpha=1$) for which in the definition of S_{ψ} in Eq. (19) $V_{\ell}=0$ and hence $S_{\psi}=\eta\langle\mathbf{J}_{\text{CD}} \cdot \mathbf{B}\rangle/\langle\mathbf{B} \cdot \nabla\zeta\rangle$. Abbreviated statements of the three puzzles and their resolution are:

Puzzle 1: *Conservation? How can the conservation of the canonical toroidal angular momentum, which is a constant of collisionless particle motion, be reconciled with the radial diffusion of guiding centers at the magnetic field diffusion rate in the paleoclassical model? Two subquestions are posed:*

(1) *Where do collision effects enter the derivation of Eq. (91) in Ref. 2, which are the Fokker-Planck coefficients for guiding center diffusion with the magnetic field diffusivity D_{η} ?*

Collisional effects are introduced into the paleoclassical model through the equilibrium ($\partial/\partial t < \nu_e$, Maxwellian-averaged electron momentum collisional relaxation rate) parallel Ohm’s law which, in combination with Faraday’s law, yields the diffusion equation for the poloidal flux in Eq. (16). Thus, as indicated in the validity condition (16) in Ref. 3, the Fokker-Planck coefficients and modified drift-kinetic equation (MDKE) that includes $\mathcal{D}\{f\}$ on its right-hand side, as in Eq. (15) above, are valid for all time scales longer than the average collision time, i.e., $t > 1/\nu_e$. The validity time scale of the MDKE is not limited by the global magnetic field diffusion time $\tau_{\eta} \sim a^2/6D_{\eta}$ as implied (incorrectly) in Ref. 3, because the $\mathcal{D}\{f\}$ Fokker-Planck operator preserves the correct geometrical properties of the diffusion process.

In most tokamak plasmas the average electron collision time is longer than the oscillatory gyro and bounce time scale motion about and relative to a poloidal flux surface. Hence, for time scales longer than $1/\nu_e$ the bounce-average guiding center ψ position (and p_{ζ}) obeys a diffusion equation like Eq. (10), has a probability distribution like that in Eq. (11), and diffuses radially [see Eqs. (10) and (13) in Ref. 3]. Thus, even a very “collisionless” particle (e.g., a relativistic electron) is subject to the paleoclassical radial diffusion process on time scales longer than the average electron collision time $1/\nu_e$, because the diffusion results from a coordinate transformation (to a coordinate system that particles oscillate about and which is locally diffusing radially with the magnetic field diffusivity D_{η}), instead of from a direct collisional modification of the particle trajectory.

(2) *Since the derivation of the canonical toroidal angular momentum p_{ζ} from $m_e d\mathbf{v}/dt = q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + \delta\mathbf{F}$ only depends on the particle velocity \mathbf{v} , the \mathbf{E} and \mathbf{B} fields and a velocity-dependent stochastic collisional force $\delta\mathbf{F}(\mathbf{v})$, how can there be dissipation of p_{ζ} , with the additive electron motion off a flux surface being controlled by the rate of magnetic field diffusion and independent of the particle velocity \mathbf{v} , as in the paleoclassical model?*

While in steady-state the \mathbf{E} and \mathbf{B} fields are “static,” a tokamak has a driven-dissipative poloidal magnetic field.^{2,3}

Its poloidal field system is not an isolated, conservative system on the magnetic field diffusion time scale. Hence, while the particle trajectories are governed by Hamilton's equations and the canonical toroidal angular momentum is a constant of the motion on the faster gyro and bounce time scales, this is not guaranteed on the longer magnetic field diffusion time scale.

As discussed in the derivation in Eqs. (4)–(15), the key hypothesis of the paleoclassical model is derived by transforming from laboratory to ψ coordinates. In laboratory coordinates the guiding center equation of motion derived from $m_e d\mathbf{v}/dt = q_e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ is $d\mathbf{x}_g/dt|_{\mathbf{x}} = \mathbf{v}_{\parallel} + \mathbf{v}_D$. Transforming to poloidal magnetic flux coordinates³ yields $d\mathbf{x}_g/dt|_{\psi} = d\mathbf{x}_g/dt|_{\mathbf{x}} + \partial\psi/\partial t|_{\mathbf{x}}(\partial\mathbf{x}_g/\partial\psi)$. Using the equation for $\partial\psi/\partial t|_{\mathbf{x}}$ from Eq. (16) yields the same equations as the characteristic curves of the transformed drift-kinetic equation.³ Thus, performing an analysis similar to that in Eqs. (7)–(10) above yields an equation for the guiding center motion in poloidal flux coordinates,

$$\frac{d\mathbf{x}_g}{dt}\Big|_{\psi} = \mathbf{v}_{\parallel} + \mathbf{v}_D + \left(\bar{D}_{\eta} \frac{\partial^2 \psi_g}{\partial x^2} - \bar{u}_G \frac{\partial \psi_g}{\partial \rho} \right) \mathbf{e}_{\psi}, \quad (21)$$

in which³ $x \equiv \rho - \rho_0$ is a local radial variable in terms of a radial coordinate ρ based on the toroidal magnetic flux and $\mathbf{e}_{\psi} \equiv \partial\mathbf{x}/\partial\psi = \nabla\theta \times \nabla\zeta / (\nabla\psi \cdot \nabla\theta \times \nabla\zeta)$ is the covariant base vector in the ψ direction. This guiding center equation yields the same poloidal (θ) and toroidal (ζ) motion as usual. However, its radial ($\nabla\psi \cdot$) component yields both the usual oscillatory radial drift motion (due to $\mathbf{v}_D \cdot \nabla\psi$) relative to and about a ψ surface, and radial diffusion of the poloidal flux guiding center position ψ_g , which indicates diffusion of the electron guiding centers and, since to lowest order $p_{\zeta} \approx -q_e \psi_g$, diffusion of p_{ζ} with the magnetic field diffusivity D_{η} .³

Puzzle 2: *Orbit dependence on η , uniqueness? How can the fact that the particle orbits depend only on the static \mathbf{E} and \mathbf{B} in a NISS plasma be reconciled with the radial diffusion of the particle guiding centers with the magnetic field diffusivity? Two subquestions are posed:*

(1) *How can the particle orbits be influenced by the resistivity η , in apparent contradiction to their basic equation of motion depending only on the static \mathbf{E} and \mathbf{B} fields?*

As indicated in Eq. (21), in poloidal magnetic flux coordinates the guiding centers diffuse radially at a rate proportional to the magnetic field diffusivity $D_{\eta} \equiv \eta/\mu_0$. This radial diffusion results from transforming the guiding center equations from laboratory to poloidal magnetic flux coordinates about which particles oscillate and that are locally diffusing radially.

(2) *How can different transport solutions with “identical equilibria” having the same \mathbf{E} and \mathbf{J}_{\parallel} but different resistivity lead to different predictions for the rate of paleoclassical radial diffusion?*

Diffusion of poloidal magnetic flux in Eq. (16) is induced by $D_{\eta} \Delta^+ \psi = \eta(\mathbf{J} \cdot \mathbf{B}) / (\langle \mathbf{B} \cdot \nabla \zeta \rangle)$. Thus, even with con-

stant \mathbf{J}_{\parallel} , diffusion of the magnetic field and hence of ψ_g and p_{ζ} are all proportional to the resistivity η .

Puzzle 3: *Uniqueness of poloidal field lines, flux? Magnetic field lines do not have a physical identity that survives from one instant to the next. However, a velocity field $\mathbf{v}_{f.l.}$, which represents the velocity of a magnetic field line but is not unique, can be ascribed to them. In NISS plasmas the magnetic flux and field lines need not move together. And in current-carrying resistive plasmas there may be no permissible $\mathbf{v}_{f.l.}$. Two subquestions are posed:*

(1) *Is the definition of the object to which electrons are tied in the paleoclassical model then problem-dependent?*

The salient properties of magnetic field lines and flux surfaces in ideal MHD and with resistivity have been explained in the context of the paleoclassical model in the first and last paragraphs in Sec. V of Ref. 2. The discussions in Ref. 2 are consistent with the discussion in the paragraph about Puzzle 3 in the Comment,¹ but go beyond the discussion there for cases with resistivity and consequently radial diffusion of poloidal magnetic flux.

The object to which electrons are tied in the paleoclassical model is the thin annulus of local poloidal magnetic flux $\delta\psi = \psi(x') - \psi(x_{00}) \approx (x' - x_{00}) \partial\psi/\partial x|_{x_{00}}$. There is no dependence on the problem of interest or ambiguity, except where $\partial\psi/\partial x = 0$ and modifications are needed to take account of this singular case, which only occurs in standard tokamak plasmas at a divertor separatrix.

(2) *For a problem in which permissible flux-conserving $\mathbf{v}_{f.l.}$ exist and are not unique, what is the basis for selecting which among them to take for the paleoclassical hypothesis?*

An initially localized thin annulus of poloidal flux $\delta\psi$ advects with the radial “grid velocity” \bar{u}_G ($\propto \langle \mathbf{E} \cdot \mathbf{B}_{pol} \rangle$) and diffuses radially as indicated in Eqs. (76)–(79) in Ref. 2. In the paleoclassical model this causes electron guiding centers to advect and diffuse radially according to Eq. (21). Thus, the radial (ψ) position of the guiding center assumes a probability distribution [see Eq. (13) in Ref. 3] like that in Eq. (11) here, which indicates the guiding center advects with \bar{u}_G and diffuses with a radial spread (variance) of order $(D_{\eta} t)^{1/2}$ that increases with time t . There is no need to define a $\mathbf{v}_{f.l.}$ for the paleoclassical model; hence there is no problem with its possible lack of uniqueness.

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¹L. L. LoDestro, Phys. Plasmas **14**, 104701 (2007).

²J. D. Callen, Phys. Plasmas **12**, 092512 (2005).

³J. D. Callen, Phys. Plasmas **14**, 040701 (2007).

⁴R. D. Hazeltine and F. L. Waelbroeck, *The Framework of Plasma Physics* (Persus, Reading, 1998), see pp. 23, 24 for the method of averaging procedure.