Appendix Z

Useful Formulas

Key Vector Relations

$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}, \quad \mathbf{A}\times\mathbf{B} = -\,\mathbf{B}\times\mathbf{A}, \quad \mathbf{A}\times\mathbf{A} = \mathbf{0},$$

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\,(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}\,(\mathbf{A}\cdot\mathbf{B}), \quad \textit{bac-cab}\text{ rule}$$

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D}) = (\mathbf{A}\cdot\mathbf{C})\,(\mathbf{B}\cdot\mathbf{D}) - (\mathbf{A}\cdot\mathbf{D})\,(\mathbf{B}\cdot\mathbf{C})$$

$$(\mathbf{A}\times\mathbf{B})\times(\mathbf{C}\times\mathbf{D}) = \mathbf{C}\,(\mathbf{A}\times\mathbf{B}\cdot\mathbf{D}) - \mathbf{D}\,(\mathbf{A}\times\mathbf{B}\cdot\mathbf{C})$$

$$\mathbf{A} = A_\parallel\hat{\mathbf{b}} + \mathbf{A}_\perp \text{ with } \hat{\mathbf{b}} \equiv \mathbf{B}/B$$

$$A_\parallel \equiv \mathbf{B}\cdot\mathbf{A}/B = \hat{\mathbf{b}}\cdot\mathbf{A}$$

$$\mathbf{A}_\perp \equiv -\,\mathbf{B}\times(\mathbf{B}\times\mathbf{A})/B^2 = -\,\hat{\mathbf{b}}\times(\hat{\mathbf{b}}\times\mathbf{A})$$

$$\boldsymbol{\nabla}\cdot\mathbf{A} = (\mathbf{B}\cdot\boldsymbol{\nabla})\,(A_\parallel/B) + (A_\parallel/B)\,(\boldsymbol{\nabla}\cdot\mathbf{B}) + \boldsymbol{\nabla}\cdot\mathbf{A}_\perp$$

$$\boldsymbol{\nabla}\cdot\mathbf{A}_\perp = -\,\mathbf{A}_\perp\cdot[\,\boldsymbol{\nabla}\ln B + (\hat{\mathbf{b}}\cdot\boldsymbol{\nabla})\hat{\mathbf{b}}\,] + (1/B)\,\hat{\mathbf{b}}\cdot\boldsymbol{\nabla}\times(\mathbf{B}\times\mathbf{A}_\perp)$$

$$\text{For } \mathbf{A}_\perp = \mathbf{B}\times\boldsymbol{\nabla}f/B^2, \quad \hat{\mathbf{b}}\cdot\boldsymbol{\nabla}\times(\mathbf{B}\times\mathbf{A}_\perp) = (\hat{\mathbf{b}}\cdot\boldsymbol{\nabla}f)\,(\hat{\mathbf{b}}\cdot\boldsymbol{\nabla}\times\hat{\mathbf{b}})$$

$$\boldsymbol{\nabla}(fg) = g\boldsymbol{\nabla}f + f\boldsymbol{\nabla}g \qquad \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}f \equiv \nabla^2 f$$

$$\boldsymbol{\nabla}\cdot(f\mathbf{A}) = \boldsymbol{\nabla}f\cdot\mathbf{A} + f\boldsymbol{\nabla}\cdot\mathbf{A} \qquad \boldsymbol{\nabla}\times\boldsymbol{\nabla}f = \mathbf{0}$$

$$\boldsymbol{\nabla}\times(f\mathbf{A}) = \boldsymbol{\nabla}f\times\mathbf{A} + f\boldsymbol{\nabla}\times\mathbf{A} \qquad \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\mathbf{A} \equiv \nabla^2\mathbf{A}$$

$$\boldsymbol{\nabla}\cdot(f\mathsf{T}) = \boldsymbol{\nabla}f\cdot\mathsf{T} + f\boldsymbol{\nabla}\cdot\mathsf{T} \qquad = \boldsymbol{\nabla}\,(\boldsymbol{\nabla}\cdot\mathbf{A}) - \boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{A})$$

$$\boldsymbol{\nabla}\times(f\mathsf{T}) = \boldsymbol{\nabla}f\times\mathsf{T} + f\boldsymbol{\nabla}\times\mathsf{T} \qquad \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}\times\mathbf{A} = 0$$

$$(\mathbf{B}\cdot\boldsymbol{\nabla})(\mathbf{A}\cdot\mathbf{C}) = \mathbf{C}\cdot(\mathbf{B}\cdot\boldsymbol{\nabla})\,\mathbf{A} + \mathbf{A}\cdot(\mathbf{B}\cdot\boldsymbol{\nabla})\,\mathbf{C}$$

$$\boldsymbol{\nabla}(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\boldsymbol{\nabla}\times\mathbf{B}) + \mathbf{B}\times(\boldsymbol{\nabla}\times\mathbf{A}) + (\mathbf{A}\cdot\boldsymbol{\nabla})\,\mathbf{B} + (\mathbf{B}\cdot\boldsymbol{\nabla})\,\mathbf{A}$$

$$\boldsymbol{\nabla}\cdot(\mathbf{A}\mathbf{B}) = \mathbf{B}\,(\boldsymbol{\nabla}\cdot\mathbf{A}) + (\mathbf{A}\cdot\boldsymbol{\nabla})\,\mathbf{B}$$

$$\boldsymbol{\nabla}\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\boldsymbol{\nabla}\times\mathbf{A} - \mathbf{A}\cdot\boldsymbol{\nabla}\times\mathbf{B}$$

$$\boldsymbol{\nabla}\times(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\,(\boldsymbol{\nabla}\cdot\mathbf{B}) - \mathbf{B}\,(\boldsymbol{\nabla}\cdot\mathbf{A}) + (\mathbf{B}\cdot\boldsymbol{\nabla})\,\mathbf{A} - (\mathbf{A}\cdot\boldsymbol{\nabla})\,\mathbf{B}$$

For the general coordinate $\mathbf{x} \equiv x\hat{\mathbf{e}}_{\rm x} + y\hat{\mathbf{e}}_{\rm y} + z\hat{\mathbf{e}}_{\rm z}$ and $|\mathbf{x}| \equiv \sqrt{x^2+y^2+z^2}$,

$$\boldsymbol{\nabla}\cdot\mathbf{x} = 3, \quad \boldsymbol{\nabla}\times\mathbf{x} = \mathbf{0}, \quad \boldsymbol{\nabla}\mathbf{x} = \mathsf{I}, \quad \boldsymbol{\nabla}\cdot\mathsf{I} = \mathbf{0}, \quad \boldsymbol{\nabla}\times\mathsf{I} = \mathbf{0}, \quad \mathbf{A}\cdot\mathsf{I} = \mathbf{A},$$

$$\boldsymbol{\nabla}|\mathbf{x}| = \mathbf{x}/|\mathbf{x}|, \quad \boldsymbol{\nabla}(1/|\mathbf{x}|) = -\,\mathbf{x}/|\mathbf{x}|^3, \quad \nabla^2(1/|\mathbf{x}|) = -\,4\pi\delta(\mathbf{x}), \quad \mathsf{I}\cdot\mathbf{A} = \mathbf{A}.$$

For a volume V enclosed by a closed, continuous surface S,

$$\int_V d^3x\ \boldsymbol{\nabla}\cdot\mathbf{A} = \oiint_S d\mathbf{S}\cdot\mathbf{A}, \quad \text{divergence, Gauss' theorem.}$$

For an open surface S bounded by a closed, continuous contour C,

$$\iint_S d\mathbf{S}\cdot\boldsymbol{\nabla}\times\mathbf{A} = \oint_C d\boldsymbol{\ell}\cdot\mathbf{A}, \quad \text{Stokes' theorem.}$$

Explicit Forms Of Vector Differentiation Operators

(for orthogonal curvilinear coordinates u^i, $\hat{\mathbf{e}}_i \equiv \boldsymbol{\nabla} u^i/|\boldsymbol{\nabla} u^i|$, $A_i \equiv \hat{\mathbf{e}}_i \cdot \mathbf{A}$)

Cartesian coordinates: $u^i = (x, y, z)$, $\int d^3x = \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \int_{-\infty}^{\infty} dz$,

$$
\begin{aligned}
\boldsymbol{\nabla} f &= \hat{\mathbf{e}}_{\rm x}\frac{\partial f}{\partial x} + \hat{\mathbf{e}}_{\rm y}\frac{\partial f}{\partial y} + \hat{\mathbf{e}}_{\rm z}\frac{\partial f}{\partial z} \\
\boldsymbol{\nabla}\cdot\mathbf{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\
\boldsymbol{\nabla}\times\mathbf{A} &= \hat{\mathbf{e}}_{\rm x}\left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right] + \hat{\mathbf{e}}_{\rm y}\left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right] + \hat{\mathbf{e}}_{\rm z}\left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right] \\
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}
\end{aligned}
$$

Cylindrical coordinates: $u^i = (r, \theta, z)$, $\int d^3x = \int_0^{\infty} r\,dr \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dz$,
with $r \equiv \sqrt{x^2+y^2}$, $\theta \equiv \arctan(y/x)$, $z \equiv z$,
and inverse relations $x = r\sin\theta$, $y = r\cos\theta$, $z = z$,

$$
\begin{aligned}
\boldsymbol{\nabla} f &= \hat{\mathbf{e}}_{\rm r}\frac{\partial f}{\partial r} + \hat{\mathbf{e}}_{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\mathbf{e}}_{\rm z}\frac{\partial f}{\partial z} \\
\boldsymbol{\nabla}\cdot\mathbf{A} &= \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z} \\
\boldsymbol{\nabla}\times\mathbf{A} &= \hat{\mathbf{e}}_{\rm r}\left[\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right] + \hat{\mathbf{e}}_{\theta}\left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right] \\
&\quad + \hat{\mathbf{e}}_{\rm z}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right] \\
\nabla^2 f &= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}
\end{aligned}
$$

Spherical coordinates: $u^i = (r, \vartheta, \varphi)$, $\int d^3x = \int_0^{\infty} r^2\,dr \int_0^{\pi} d\vartheta\,\sin\vartheta \int_0^{2\pi} d\varphi$,
with $r \equiv \sqrt{x^2+y^2+z^2}$, $\vartheta \equiv \arctan(\sqrt{x^2+y^2}/r)$, $\varphi \equiv \arctan(y/x)$,
and inverse relations $x = r\sin\vartheta\cos\varphi$, $y = r\sin\vartheta\sin\varphi$, $z = r\cos\vartheta$,

$$
\begin{aligned}
\boldsymbol{\nabla} f &= \hat{\mathbf{e}}_{\rm r}\frac{\partial f}{\partial r} + \hat{\mathbf{e}}_{\vartheta}\frac{1}{r}\frac{\partial f}{\partial \vartheta} + \hat{\mathbf{e}}_{\varphi}\frac{1}{r\sin\vartheta}\frac{\partial f}{\partial \varphi} \\
\boldsymbol{\nabla}\cdot\mathbf{A} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\vartheta}\frac{\partial}{\partial \vartheta}(\sin\vartheta A_\vartheta) + \frac{1}{r\sin\vartheta}\frac{\partial A_\varphi}{\partial \varphi} \\
\boldsymbol{\nabla}\times\mathbf{A} &= \hat{\mathbf{e}}_{\rm r}\frac{1}{r\sin\vartheta}\left[\frac{\partial}{\partial \vartheta}(\sin\vartheta A_\varphi) - \frac{\partial A_\vartheta}{\partial \varphi}\right] \\
&\quad + \hat{\mathbf{e}}_{\vartheta}\left[\frac{1}{r\sin\vartheta}\frac{\partial A_r}{\partial \varphi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi)\right] + \hat{\mathbf{e}}_{\varphi}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\vartheta) - \frac{\partial A_r}{\partial \vartheta}\right] \\
\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial \vartheta}\left(\sin\vartheta\frac{\partial f}{\partial \vartheta}\right) + \frac{1}{r^2\sin^2\vartheta}\frac{\partial^2 f}{\partial \varphi^2}
\end{aligned}
$$

Physical Constants

m_e	electron mass	9.11×10^{-31} kg, 511 keV
m_p	proton mass	1.67×10^{-27} kg, 938 MeV
m_p/m_e	mass ratio	$1836 = (42.85)^2$
e	elementary charge	1.602×10^{-19} C (= J/eV)
c	speed of light in vacuum	3×10^8 m/s $= 1/\sqrt{\mu_0 \epsilon_0}$
μ_0	permeability of vacuum	$4\pi \times 10^{-7}$ N/A^2
ϵ_0	permittivity of vacuum	8.85×10^{-12} F/m, $4\pi\epsilon_0 \simeq 10^{-10}$
h	Planck constant	6.626×10^{-34} J$\cdot$s
N_A	Avogadro constant	6.022×10^{23}#/mol
e/k_B	Boltzmann constant	11 604 K/eV

Key Plasma Formulas

Quantities are in SI (mks) units except temperature and energy which are expressed in eV; Z_i is the ion charge state; $A_i \equiv m_i/m_p$ is the atomic mass number.

Frequencies

electron plasma $$\omega_{pe} \equiv \sqrt{\frac{n_e e^2}{m_e \epsilon_0}} \simeq 56\sqrt{n_e} \text{ rad/s}, \quad f_{pe} \simeq 9\sqrt{n_e} \text{ Hz}$$

ion gyrofrequency $$\omega_{ci} \equiv \frac{q_i B}{m_i} \simeq 0.96 \times 10^8 \frac{Z_i B}{A_i} \text{ rad/s}$$

electron collision $$\nu_e \equiv \frac{4}{3\sqrt{\pi}} \nu(v_{Te}) \simeq \frac{5 \times 10^{-11} n_e Z_i}{[T_e(\text{eV})]^{3/2}} \left(\frac{\ln \Lambda}{17}\right) \text{ s}^{-1}$$

Lengths

electron Debye $$\lambda_{De} \equiv \sqrt{\frac{\epsilon_0 T_e}{n_e e^2}} \simeq 7.4 \times 10^3 \sqrt{\frac{T_e(\text{eV})}{n_e}} \text{ m}$$

ion gyroradius $$\varrho_i \equiv \frac{v_{Ti}}{\omega_{ci}} \simeq 1.4 \times 10^{-4} \frac{\sqrt{T_i(\text{eV})\, A_i}}{Z_i B} \text{ m}$$

electron collision $$\lambda_e = \frac{v_{Te}}{\nu_e} \simeq 1.2 \times 10^{16} \frac{[T_e(\text{eV})]^2}{n_e Z_i} \left(\frac{17}{\ln \Lambda}\right) \text{ m}$$

Speeds, Velocities

electron thermal $$v_{Te} \equiv \sqrt{2\, T_e/m_e} \simeq 5.9 \times 10^5 \sqrt{T_e(\text{eV})} \text{ m/s}$$

ion thermal $$v_{Ti} \equiv \sqrt{2\, T_i/m_i} \simeq 1.4 \times 10^4 \sqrt{T_i(\text{eV})/A_i} \text{ m/s}$$

ion acoustic ($T_e >> T_i$) $$c_S \equiv \sqrt{Z_i\, T_e/m_i} \simeq 10^4 \sqrt{Z_i\, T_e(\text{eV})/A_i} \text{ m/s}$$

Alfvén $$c_A \equiv B/\sqrt{\mu_0\, \rho_m} \simeq 2.2 \times 10^{16} B/\sqrt{n_i\, A_i} \text{ m/s}$$

electron diamagnetic flow ($dT_e/dx = 0$) $$\mathbf{V}_{*e} \equiv \frac{T_e}{q_e B}\left(\frac{1}{n_e}\frac{dn_e}{dx}\right)\hat{\mathbf{e}}_y = \frac{T_e(\text{eV})}{B\, L_n}\hat{\mathbf{e}}_y \text{ m/s}$$

electron drift in $B(x)$ (average, low β) $$\bar{\mathbf{v}}_{de} = \frac{2T_e}{q_e B}\left(\frac{1}{B}\frac{dB}{dx}\right)\hat{\mathbf{e}}_y = -\frac{2\, T_e(\text{eV})}{B\, L_B}\hat{\mathbf{e}}_y \text{ m/s}$$

Drift, flow velocities (for $\varrho\boldsymbol{\nabla}_\perp << 1$, $\omega << \omega_c$) **perpendicular to B:**

particle drift velocities		plasma species flow velocities
$\mathbf{v}_F = \mathbf{F}\times\mathbf{B}/qB^2$	general force	$\mathbf{V}_F = \bar{\mathbf{F}}\times\mathbf{B}/qB^2$
$\mathbf{v}_E = \mathbf{E}\times\mathbf{B}/B^2$	$\mathbf{E}\times\mathbf{B}$	$\mathbf{V}_E = \mathbf{E}\times\mathbf{B}/B^2$
$\mathbf{v}_\mu = \mathbf{B}\times\mu\boldsymbol{\nabla}B/qB^2$	μ grad-B	
$\mu \equiv mv_\perp^2/2B$		
$\mathbf{v}_\kappa = \mathbf{B}\times mv_\parallel^2\boldsymbol{\kappa}/qB^2$	curvature	
$\boldsymbol{\kappa} \equiv (\hat{\mathbf{b}}\cdot\boldsymbol{\nabla})\hat{\mathbf{b}} = -\mathbf{R}_C/R_C^2$		
	diamagnetic	$\mathbf{V}_* = \mathbf{B}\times\boldsymbol{\nabla}p/nqB^2$
$\mathbf{v}_p = \mathbf{B}\times m(d\mathbf{v}_d/dt)/qB^2$	polarization	$\mathbf{V}_p = \mathbf{B}\times m(d\mathbf{V}/dt)/qB^2$
	friction	$\mathbf{V}_\eta = \mathbf{R}\times\mathbf{B}/nqB^2$
	viscosity	$\mathbf{V}_\pi = \mathbf{B}\times(\boldsymbol{\nabla}\cdot\boldsymbol{\pi})/nqB^2$
$\mathbf{v}_{d\perp} = \mathbf{v}_E + \mathbf{v}_\mu + \mathbf{v}_\kappa + \mathbf{v}_p$	total	$\mathbf{V} = \mathbf{V}_E + \mathbf{V}_* + \mathbf{V}_p + \mathbf{V}_\eta + \mathbf{V}_\pi$

Diffusivities

no magnetic field
$$\nu_e\lambda_e^2 \equiv v_{Te}^2/\nu_e \simeq 7\times10^{21}\,\frac{[T_e(\text{eV})]^{5/2}}{n_e\,Z_i}\left(\frac{17}{\ln\Lambda}\right)\ \text{m}^2/\text{s}$$

magnetic field
$$\eta/\mu_0 \equiv (m_e\nu_e/n_e e^2)/\mu_0 = \nu_e(c/\omega_{pe})^2 \simeq 1.4\times10^3\left(\frac{Z_i}{[T_e(\text{eV})]^{3/2}}\right)\left(\frac{\ln\Lambda}{17}\right)\ \text{m}^2/\text{s}$$

classical
$$\nu_e\varrho_e^2 = \beta_e\,(\eta/\mu_0) \simeq 5.6\times10^{-22}\,\frac{n_e\,Z_i}{B^2\,[T_e(\text{eV})]^{1/2}}\left(\frac{\ln\Lambda}{17}\right)\ \text{m}^2/\text{s}$$

Dimensionless

number of electrons in Debye cube
$$n_e\lambda_{De}^3 \simeq 4.1\times10^{11}\,[T_e(\text{eV})]^{3/2}/\sqrt{n_e}$$

Coulomb logarithm
$$\ln\Lambda \equiv \ln\left(\frac{\lambda_D}{\max\{\,b_{\min}^{\text{cl}}, b_{\min}^{\text{qm}}\,\}}\right)$$
$$b_{\min}^{\text{cl}} = Z_i/(12\pi n_e\lambda_{De}^2) \simeq 5\times10^{-10} Z_i/T_e(\text{eV})\ \text{m}$$
$$b_{\min}^{\text{qm}} = h/(4\pi m_e v) \simeq 1.1\times10^{-10}/[T_e(\text{eV})]^{1/2}\ \text{m}$$

plasma to magnetic pressure
$$\beta \equiv \frac{P}{B^2/2\mu_0} = \frac{n_e\,T_e + n_i\,T_i}{B^2/2\mu_0} \simeq 4.0\times10^{-25}\left(\frac{n_e}{B^2}\right)\left[T_e(\text{eV}) + \frac{n_i}{n_e}\,T_i(\text{eV})\right]$$

Lundquist number
$$S \equiv \frac{a^2/(\eta/\mu_0)}{L_\parallel/c_A} \simeq 1.6\times10^{13}\,\frac{a^2 B\,[T_e(\text{eV})]^{3/2}}{L_\parallel\,Z_i\,\sqrt{n_i\,A_i}}\left(\frac{17}{\ln\Lambda}\right)$$

Fundamental Equations of Physics

Mechanics

$m\,\mathbf{a}$	$\equiv$	$m\,d\mathbf{v}/dt = \mathbf{F}, \quad \mathbf{v} \equiv d\mathbf{x}/dt$	Newton's second law
$\mathbf{F}$	$=$	$q\,(\mathbf{E} + \mathbf{v}\times\mathbf{B})$	Lorentz force
H	$=$	$\lvert\mathbf{p} - q\mathbf{A}\rvert^2/2m + q\,\phi, \quad \mathbf{p} = m\mathbf{v} + q\mathbf{A}$	Hamiltonian, energy
$d\mathbf{p}/dt$	$=$	$-\,\partial H/\partial\mathbf{q}, \;\; d\mathbf{q}/dt = \partial H/\partial\mathbf{p}$	Hamilton's equations

Electrodynamics

$\nabla\cdot\mathbf{E}$	$=$	ρ_q/ϵ_0	Gauss's law
$\nabla\times\mathbf{E}$	$=$	$-\,\partial\mathbf{B}/\partial t$	Faraday's law
$\nabla\cdot\mathbf{B}$	$=$	0	no magnetic monopoles
$\nabla\times\mathbf{B}$	$=$	$\mu_0\,(\mathbf{J} + \epsilon_0\,\partial\mathbf{E}/\partial t)$	Ampere's law, $\mu_0\epsilon_0 = 1/c^2$
0	$=$	$\partial\rho_q/\partial t + \nabla\cdot\mathbf{J}$	charge continuity equation
$\mathbf{E}$	$=$	$-\nabla\phi - \partial\mathbf{A}/\partial t, \;\; \mathbf{B} = \nabla\times\mathbf{A}$	potential representations

Plasma Physics

Plasma kinetic equation (PKE) for distribution function $f \equiv f_s(\mathbf{x}, \mathbf{v}, t)$:

$$\partial f/\partial\,t + \mathbf{v}\cdot\partial f/\partial\,\mathbf{x} + (q/m)\,(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\partial f/\partial\,\mathbf{v} = \mathcal{C}\{f\}.$$

Density, flow moments and charge, current densities:

$$n_s \equiv \int d^3v\, f_s, \quad \mathbf{V}_s \equiv \int d^3v\,\mathbf{v}\,f_s/n_s, \quad \rho_q \equiv \textstyle\sum_s n_s q_s, \quad \mathbf{J} \equiv \textstyle\sum_s n_s q_s \mathbf{V}_s.$$

Gibb's (A: adiabatic) distribution of plasma species with temperature T:

$$f_A = n_0\left(\frac{m}{2\pi T}\right)^{3/2} e^{-H/T}; \quad n_A(\mathbf{x}, t) = n_0 e^{-q\phi/T}, \quad \text{Boltzmann relation.}$$

Maxwellian (M: collisional equilibrium) distribution ($v_T \equiv \sqrt{2T/m}$):

$$f_M = n\left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(-\frac{m\lvert\mathbf{v}'\rvert^2}{2T}\right) = \frac{n\,e^{-v'^2/v_T^2}}{\pi^{3/2}v_T^3}, \quad \mathbf{v}' \equiv \mathbf{v} - \mathbf{V}.$$

Species fluid moment equations (density, momentum, energy):

$$\partial n/\partial t + \nabla\cdot n\mathbf{V} = 0, \qquad nT \equiv \int d^3v\,(mv'^2/3)\,f,$$

$$mn\,(d\mathbf{V}/dt) = nq\,(\mathbf{E} + \mathbf{V}\times\mathbf{B}) - \nabla p - \nabla\cdot\boldsymbol{\pi} + \mathbf{R}, \qquad d/dt \equiv \partial/\partial t + \mathbf{V}\cdot\nabla,$$

$$(3/2)(n\,dT/dt) + p\,\nabla\cdot\mathbf{V} = -\nabla\cdot\mathbf{q} - \boldsymbol{\pi} : \nabla\mathbf{V} + Q, \qquad p \equiv nT.$$

Magnetohydrodynamics (plasma fluid description, isotropic pressure and isentropic responses for plasma species, $\rho_m \equiv \sum_s n_s m_s, \mathbf{V} \equiv \sum_s n_s m_s \mathbf{V}_s/\rho_m$):

$$\partial\rho_m/\partial t + \nabla\cdot\rho_m\mathbf{V} = 0, \qquad \mathbf{E} + \mathbf{V}\times\mathbf{B} = \eta\mathbf{J}, \qquad P \equiv \textstyle\sum_s p_s,$$

$$\rho_m(d\mathbf{V}/dt) = \mathbf{J}\times\mathbf{B} - \nabla P, \quad d\ln\left(P/\rho_m^\Gamma\right)/dt = (\Gamma - 1)\,\eta J^2/P \simeq 0.$$