Fundamentals of
Plasma Physics

James D. Callen
University of Wisconsin, Madison

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PREFACE

Plasma physics is a relatively new branch of physics that became a mature science over the last half of the 20th century. It builds on the fundamental areas of classical physics: mechanics, electrodynamics, statistical mechanics, kinetic theory of gases, and fluid mechanics. The distinguishing feature of the plasma medium is that its properties are determined by the nature of the interactions between the charged particles in it — collective rather than binary and weak compared to their thermal motions.

The collective but weak interactions in a plasma embody many physical processes over a wide range of length and time scales: predominantly deterministic particle motion which however may be diffusive on long time scales, internal generation of microscopically irregular but macroscopically smooth electromagnetic fields, both adiabatic and inertial (or fluidlike) plasma responses, dielectric-medium-type electrical properties, and various flow regimes (laminar, transitional, shock and turbulent). These processes lead to a wide variety of interesting collective phenomena, e.g., dielectric shielding of charges, waves in the medium, transfer of energy from waves to particles (via Landau damping, a "collisionless" wave-particle resonance effect), transfer of energy from a distribution of particles into waves (instabilities), and turbulence in the six-dimensional (three real plus three velocity space coordinates) phase space.

Increased understanding of plasma physics has both been stimulated by, and paced, the development of its many important applications, e.g., magnetic and inertial approaches to fusion, space and astrophysical plasmas, plasma processing of materials, and coherent radiation generation (typically via acceleration of beams of electrons or ions). Thus, plasma physics has developed in large part as a branch of applied or engineering physics — "science with a purpose."

The primary objective of this book is to present and develop the fundamentals and principal applications of plasma physics. The emphasis is on what is usually called high-temperature plasma physics in which the plasma is nearly fully ionized and neutral particles have small effects on the plasma behavior. The level is meant to be suitable for senior undergraduate students through advanced graduate students and active researchers. Pedagogically, it begins from an elemental or microscopic description, then uses this to develop macroscopic models of plasmas, and finally uses these models to discuss practical applications. A variety of applications of plasma physics are discussed throughout the text; many others are covered in the problems at the end of each chapter. In concert with the modern trend in the physical sciences, SI (*Système International d'Unités*) or mks units are used throughout.

This book has evolved primarily from lecture notes developed while teaching various plasma physics courses at the University of Wisconsin-Madison over more than two decades (1979–2003) and in part from teaching three years at Massachusetts Institute of Technology (1969–1972). My own research and teaching has been predominantly in magnetic fusion research, which has been the dominant driving force behind the development of the science of plasma physics over this period. However, because plasma physics has grown into a mature science whose principles are broadly applicable, I attempt to develop the fundamental concepts in an application-independent manner. In addition, many different types of applications of plasma physics are discussed throughout the book.

The science of plasma physics draws heavily on many areas of classical physics and applied mathematics. Typically, not all of these subjects are well known to the wide variety of students (from physics, engineering physics, electrical engineering, nuclear engineering and other undergraduate curricula) who begin studies of plasma physics. Also, most of the needed background material is not readily available in concise, accessible forms. Thus, a number of Appendices have been written to provide relevant summaries; they give important supplementary information that is an integral part of this textbook. Finally, "Appendix Z" (to be placed on pages inside book covers) provides sets of basic formulas that are useful throughout the book — vector relations, vector differentiation operators, physical constants, and key plasma formulas.

This book is designed for teaching plasma physics at a variety of levels. (It may also serve as a useful reference book for active researchers in plasma physics.) For example, it could be used as the basis for a two (or more) semester graduate-level course on plasma physics, at the rate of approximately one chapter section per one hour lecture. However, it could also be used for teaching a fast-paced, one-semester introductory course on plasma physics by covering only the sections at the first of most of the chapters. Intermediate-level subjects that could be omitted without compromising understanding of later sections are indicated by an asterisk (*) at the end of the respective section titles. Advanced material, which is relevant mostly for research purposes, is similarly indicated by a plus sign (+). Bibliographies at the end of each of the chapters and appendices provide information on other textbooks and research literature that should be consulted for further details or supplementary course material. Individual chapters of this book will be made available (in draft form) via my public web page (`http://homepages.cae.wisc.edu/∼callen`) as soon as they are available.

The large number of problems at the end of each chapter are graduated in level of difficulty commensurate with the various levels and styles of courses that might be taught from the book. Specifically, the levels of the problems are classified according to their nature and consequent degree of difficulty: evaluational (/), application development (//) and conceptual development (///). Also, the level of material involved in solving the problem is indicated: basic (no mark), intermediate (*), or advanced (+).

(Detailed acknowledgements of help by others and assistance in the preparation of this manuscript will be written later.)

Introduction

Plasma is often called the fourth state of matter. The various states of matter occur as a substance is heated to temperatures above the binding energies for particular states of matter and thereby undergoes phase transitions. As an example, consider the states of $H_2$0 and its molecular, atomic and elementary particle constituents at various temperatures, as indicated in Fig. 1. Below 273 K ($\sim$ 0.0235 eV[1]) it is in a crystalline form known as ice — a solid, the

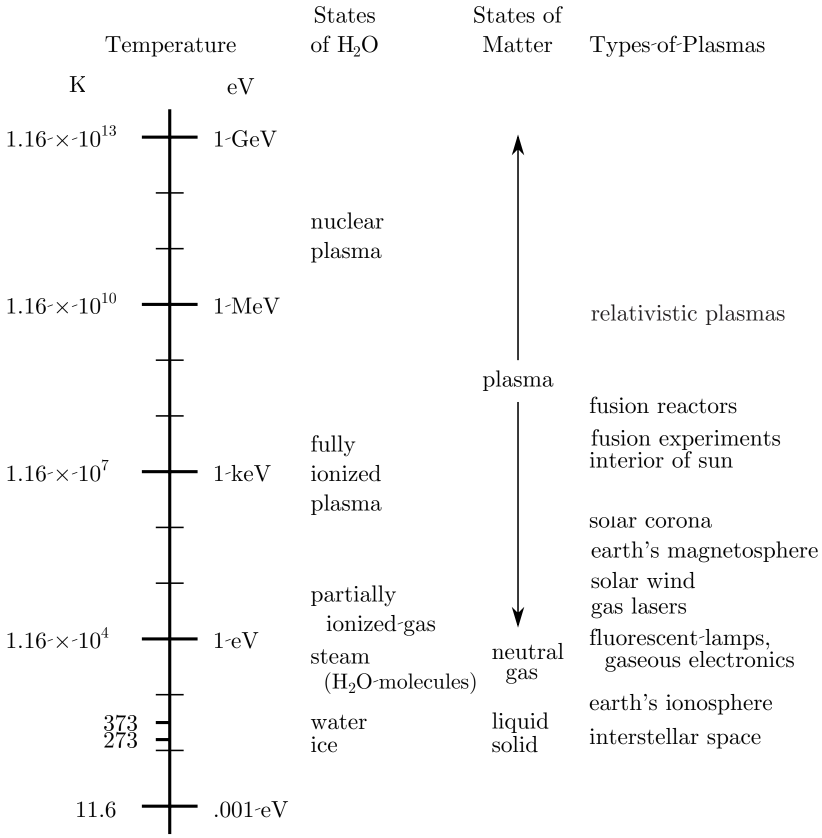

Figure 1: Schematic of states of H_2O as it is heated. Also shown are the corresponding states of matter and some of the types of plasmas that can occur in the various temperature ranges indicated.

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[1] Temperatures (and particle energies) in plasma physics are usually quoted in electron volts, abbreviated eV. The conversion factor from Kelvin to eV is Boltzmann's constant k_B divided by the electron charge: $k_B/e \simeq 1$ eV/11,604.4 K.

first state of matter, which is a strongly coupled medium (binding energy large compared to thermal energy). At temperatures between 273 K and 373 K the crystalline bonds are broken, but large scale molecular structures exist and H_20 is called water — a liquid, the second state of matter, which is also a strongly coupled medium. At temperatures above 373 K (~ 0.032 eV) the long-scale molecular structure bonds are broken and the independent H_20 molecules form a gas, which is commonly known as steam. Upon further heating to a temperature of the order of the molecular binding energy (~ 0.3 eV), the molecules dissociate into independent hydrogen and oxygen atoms. While this is no longer steam, it is still a gas in which the elemental constituents (H_2 and 0_2) are electrically neutral. This third state of matter is a neutral gas, which is a weakly coupled medium — on average, interactions between particles are weak, compared to their thermal motions.

We finally reach the plasma or fourth state of matter when we heat the gas to the point where a significant fraction of the atoms are dissociated (atomic bonds broken) into negatively charged electrons and positively charged ions to form an ionized gas. The fraction of the atoms that are dissociated is called the degree of ionization. The binding energy of the most weakly bound electron in atoms of all types is typically of the order of the 13.6 eV binding energy of an electron in the hydrogen atom. As discussed in Section 7 of Appendix A (Section A.7), when the temperature increases to a significant fraction (~ 0.02–1) of the electron binding energy, collisions between the atoms in their thermal motion cause a non-negligible fraction of the atoms to become ionized. Electron temperatures in the few eV range typically produce a partially ionized gas.

An ionized gas is in the plasma state if the charged particle interactions are predominantly collective rather than just binary. (Binary interaction collisions are one-at-a-time interactions with other individual charged particles or neutral atoms and are the dominant ones in neutral gases.) In a plasma the interactions are collective because many charged particles interact simultaneously, but weakly, through their "long-range" electromagnetic fields — in particular their Coulomb electric fields. Thus, a plasma is a collective but weakly coupled medium in which interaction energies are much smaller than thermal energies.

At temperatures above a few eV the ionization becomes essentially complete. At this point, it is an almost completely ionized gas and it is nearly always in the plasma state; hence it is then usually called a fully ionized plasma. Further heating of a collection of such particles would successively break nuclear bonds ($\sim$ MeV) and quark bonds ($\sim$ GeV). These result in nuclear and quark-gluon plasmas, respectively. However, such states are beyond the scope of normal plasma physics and will not be treated in this book.

The word plasma, which comes from the Greek $\pi\lambda\alpha\sigma\mu\alpha$, means something molded. It was introduced by Tonks and Langmuir in 1929 to describe the behavior of the ionized gas in an electrical discharge tube, which they found could be manipulated by a magnetic field. While most plasmas can indeed be manipulated by magnetic fields to some degree, their collective behavior often resembles that of an electrically charged, shapeless, structureless fluid that oozes about mostly of its own accord, as one might imagine an electrically active lump of jelly would. Thus, the name plasma is only partially appropriate — it expresses a hope but perhaps not always the reality.

Some common types of plasmas are indicated on the right side of Fig. 1. Partially ionized plasmas include various types of gas discharges (fluorescent lamps, gas lasers, arc discharges, plasmas for materials processing) and the earth's ionosphere. The earth's magnetosphere and the solar corona are prominent space physics examples of nearly fully ionized plasmas. Since most of the vastness of interstellar space is in the plasma state, it is often said that 99% of the visible universe is governed by plasma physics. (However, since the interiors of stars are also in the plasma state, the actual fraction of particles in the visible universe that are in the plasma state is much closer to unity.)

The most prominent examples of high temperature, essentially fully ionized plasmas are those in the solar wind and in fusion experiments. The latter experiments seek to confine plasmas either with magnetic fields or inertially at temperatures of about 10 keV or greater together with a product of the plasma density and the plasma confinement time of more than 10^{20} $\mathrm{m}^{-3}\cdot\mathrm{s}$. The objective of creating such plasmas is to develop an environmentally attractive new energy source based on the exothermic fusion of light ions. For example, the fusion of deuterium and tritium (isotopes of hydrogen) nuclei produces 17.59 MeV of energy, which is much larger than the 4.65 keV of collision energy that is required to overcome the Coulomb potential barrier between the charged ions. In addition to these thermal plasma examples, many types of modern devices for generating coherent radiation are governed by the collective interactions of plasma physics: free electron lasers, ion beams, relativistic electron beams, and gyrotrons.

This book concentrates on the physics of fully ionized, nonrelativistic plasmas composed of electrons and ions, which usually means temperatures and particle energies ranging from about 10 eV to 100 keV. The physics of partially ionized plasmas, which combines plasma and atomic physics, and chemistry, is covered only partially through a few examples and problems. Quantum mechanical effects are mostly neglected because, while there are various types of quantum mechanical plasmas, for the plasmas of interest here the most relevant interaction distances are usually much longer than the de Broglie wavelength.

The fundamental processes in a plasma are governed primarily by classical physics. The motion and interactions of individual charged particles are described by the usual equations of classical mechanics and electrodynamics — see Appendix A. While relativistic effects in mechanics are important for radiative processes and in very hot, "relativistic" plasmas where the electron temperature becomes a significant fraction of the electron rest mass energy (511 keV), they can mostly be neglected for the plasmas of interest here.

The distribution of the charged particles in the relevant six-dimensional phase space (three spatial and three velocity space coordinates) is governed by a plasma kinetic equation that takes account of the motion of charged particles in the extant electromagnetic fields, and of the Coulomb collisions between the charged particles in the plasma. While the velocity distribution of charged particles in a plasma is often close to the collisional equilibrium Maxwellian distribution, ordinary statistical mechanics is not usually applicable to plasmas — because collisional relaxation processes in plasmas are quite slow (compared to various physical processes in plasmas), and because plasmas are often in "unstable" and hence strongly nonequilibrium states. In unstable plasmas small perturbations grow exponentially in time by transferring energy from the charged particle distribution into collective motions of the plasma. Non-equilibrium statistical mechanics descriptions have been developed for some particular plasma situations; however, it has not been possible to give a general description of plasmas using this approach.

When the velocity distribution is close to a Maxwellian, it is often sufficient to use fluid moment descriptions (e.g., plasma density, momentum, and energy equations). Then, the description of plasmas becomes analogous to descriptions of ordinary neutral fluids. However, the effects of electromagnetic fields on the charged particles and the separate (and often different) behavior of the electron and ion components in a plasma make these fluid moment descriptions much more complicated. Nonetheless, plasmas exhibit a rich variety of the types of phenomena usually associated with neutral fluids — wave propagation, instabilities, turbulence, and turbulent transport.

This book is organized broadly as follows. Part I develops descriptions of the fundamental processes in plasmas — collective phenomena, Coulomb collisions, structure of magnetic fields, charged particle motion, and the various models [kinetic, two-fluid, and magnetohydrodynamics (MHD)] that are used to describe plasmas. Then, Part II discusses the various types of waves that occur in stable plasmas. Plasma kinetic theory and the Coulomb collision operator, and their applications are discussed in Part III. The plasma transport processes induced by Coulomb collisions in a stable plasma and their effects on fluid moment descriptions of plasmas, and on plasma confinement are discussed in Part IV. The equilibrium and stability properties of a plasma are developed in Part V. Finally, Part VI provides an introduction to nonlinear plasma theory, and to plasma turbulence and the anomalous transport it induces.