Quiz # 1 Suggested Solution

last updated: October 2, 2000

1. (30 points) base conversion, arithmetics, binary code

(a) (6 points) Convert the following decimal number to radix 5 representation

Answer: \((23.44)_{10} = (43.21)_5\). Note that \(23 = 4 \times 5 + 3\), and \(0.2 \times 5 = 1.0\).

(b) (6 points) Convert \((11.33)_{8}\) to \((A)_{16}\).

Answer: \((11.33)_{8} = (001001.01101100)_2 = (9.6C)_{16}\)

(c) (6 points) Find the radix \(r\) that satisfies the following equation:

\((16)_r \times (19)_r = (28A)_r\)

Answer: LHS (left hand side) = \((r+6)(r+9) = r^2 + 15r + 54\). RHS = \(2r^2 + 8r + 10\). This yields \(r^2 - 7r - 44 = (r - 11)(r + 4) = 0\) Thus, \(r = 11\)

(d) (6 points) Perform the following arithmetic operations with the specified radices:

Answer: \((AA)_{16} \times (BB)_{16} = (7C2E)_{16}\)

(e) (6 points) BCD Code

The result of a BCD addition is 4578. It is known that carries occurred from the positions of digit 7 and digit 5. What were the binary values of the sums before the correction including the value of the carry?

Answer: The binary value of the sums are \([0100 \ 1111 \ 10001 \ 1000]\). Note that in the digit 7, the intermediate result is 17, and in the digit 5, the intermediate result is 15!

2. (20 points) Boolean Algebra

(a) (5 points) Find the dual expression of the following Boolean identity:

\(XY + \overline{XZ} + YZ = XY + \overline{XZ}\)
Answer: \((X + Y)(\overline{X} + Z)(Y + Z) = (X + Y)(\overline{X} + Z)\)

(b) (5 points) Find the complement of the following Boolean expression

\[ AB + \overline{A} \cdot \overline{B} \]

The result must be in SOP format.

Answer: \(A\overline{B} + \overline{A}B\)

(c) (5 points) Simplify the following Boolean function into POS (product of sum) format using minimum number of literals:

\[ f(a, b, c) = \prod(2, 4, 6) \]

Answer: \(f(a, b, c) = a\overline{c} + b\overline{c}\). Thus, \(f(a, b, c) = (\overline{a} + c)(\overline{b} + c)\).

(d) (5 points) Give the Karnaugh map of the Boolean function below:

\[ g(w, x, y, z) = (\overline{w} + y + z)(w + \overline{x} + \overline{y})(x + \overline{z}) \]

Answer: Note that \(g(w, x, y, z) = w\overline{y} \cdot \overline{x} + \overline{w}xy + \overline{x}z\). Hence,

3. (15 points)

(a) (7 points) The following is a Boolean conditional statement. Determine whether this statement is TRUE or FALSE. If it is deemed as a TRUE statement, give a concise explanation. If it is deemed as a FALSE statement, give a counter example. Answer alone without explanation or a counter example will not receive ANY credit.

If \(A \cdot B = 1\), then \(\overline{AD} + \overline{BD} + AB\overline{D} + B \cdot \overline{C} + A\overline{C} \cdot \overline{D} = \overline{D}\).

Answer: [TRUE]
### Formal Algebraic Proof:

<table>
<thead>
<tr>
<th>Derivation Steps</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{AD} + \overline{BD} + AB\overline{D} + \overline{B} \cdot \overline{C} + AC \cdot \overline{D} )</td>
<td></td>
</tr>
<tr>
<td>( = (AB)\overline{D} + AB\overline{D} + \overline{B} \cdot \overline{C} + AC \cdot \overline{D} )</td>
<td>( ab + ad = a(b + d), \overline{a} + \overline{b} = \overline{a} \overline{b} )</td>
</tr>
<tr>
<td>( = 0 \cdot D + 1 \cdot \overline{D} + AB \cdot \overline{B} \cdot \overline{C} + AC \cdot \overline{D} )</td>
<td>( AB = 1, 1 \cdot x = x )</td>
</tr>
<tr>
<td>( = \overline{D} + A \cdot 0 \cdot \overline{C} + AC \cdot \overline{D} )</td>
<td>( 0 + x = x, a \cdot \overline{a} = 0 )</td>
</tr>
<tr>
<td>( = \overline{D} + 0 + AC \cdot \overline{D} )</td>
<td>( 0 \cdot x = 0 )</td>
</tr>
<tr>
<td>( = \overline{D} )</td>
<td>( a + ab = a, a + 0 = a )</td>
</tr>
</tbody>
</table>

**Informal alternate proof:** \( A \cdot B = 1 \) implies \( A = B = 1 \). Thus the statement becomes

\[
0 \cdot D + 0 \cdot D + 1 \cdot \overline{D} + 0 + 1 \cdot \overline{C} \cdot \overline{D} = \overline{D}
\]

Equivalently, we have

\[
\overline{D} + \overline{C} \cdot \overline{D} = \overline{D}
\]

Using the theorem \( X + XY = X \), above equation is proved.

(b) (8 points) Denote \( A, B \) to be known Boolean variables and \( f(A, B) \) an unknown Boolean function. Solve for **ALL distinct** solutions of the following Boolean equation

\[
(A + B) \cdot f(A, B) = B
\]

Give your answer(s) in the product of maxterm format. Hint: A truth table may help.

**Answer:** \( f(A, B) = \prod M(0, 2) \text{ or } \prod M(2) \). Refer to the truth table below:

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( A + B )</th>
<th>( f(A, B) )</th>
<th>( (A + B) \cdot f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0 or 1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

4. (15 points) **Boolean expressions**

(a) (10 points) Find the sum-of-minterm format of the boolean function

\[
F(A, B, C, D) = (A + B)(\overline{A} + C)(B + D)
\]

**Answer:** \( F(A, B, C, D) = \sum(4, 5, 6, 7, 11, 14, 15) \)

(b) (5 points) Implement

\[
g(w, x, y, z) = (w \cdot \overline{y} + x)(\overline{x} + \overline{w}z) + wx y
\]
using AND, OR, and NOT logic gates. Assume that the complements of these four Boolean variables have been made available. Do not simplify or modify above Boolean function.

**Answer:** straight forward.

5. (20 points)

(a) (10 points) If \(\overline{a} \land b \land \overline{c}\) and \(\overline{a} \cdot \overline{c} \cdot \overline{d}\) are both in the set of *essential prime implicants* (EPI) of a certain Boolean function \(f(a, b, c, d)\). Which of the following product term(s) definitely can **NOT** be a *prime implicant* (PI) of \(f(a, b, c, d)\)? (Place an X in the X box)? Explain each answer (the one you mark with X) briefly. No grade will be given without explanation. Hint: Use a Karnaugh map.

**Answer:**

<table>
<thead>
<tr>
<th>Product Term</th>
<th>X</th>
<th>Brief Explanations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(bc\overline{d})</td>
<td></td>
<td>not covering (m_0) nor (m_5)</td>
</tr>
<tr>
<td>(\overline{b}c)</td>
<td>X</td>
<td>will render (\overline{a} \cdot \overline{c} \cdot \overline{d}) non EPI</td>
</tr>
<tr>
<td>(\overline{a}bd)</td>
<td></td>
<td>not adjacent to any product term</td>
</tr>
<tr>
<td>(\overline{abc})</td>
<td>X</td>
<td>will render (\overline{ab}\overline{c}) non-PI</td>
</tr>
<tr>
<td>(abc)</td>
<td></td>
<td>not adjacent to any product term</td>
</tr>
</tbody>
</table>

(b) (10 points) Simplify the following Boolean function with don’t care terms into SOP (sum of product) format using minimum number of literals:

\[
f(w, x, y, z) = \sum m(2, 3, 4, 5, 6, 8, 11), \quad d(w, x, y, z) = \sum m(7, 9, 14)
\]

**Answer:** There are two possible answers:

\[
f(w, x, y, z) = \overline{wx} + \overline{wy} + w\overline{x} \cdot \overline{y} + \left\{ \begin{array}{c} \overline{wx} z \\ \overline{x} y z \end{array} \right\}
\]

(1)