1. (20 points) number representations and conversion
   (a) (4 points) \((CBA)_{16} = (\quad)_{10}\)
      \textbf{Answer:} integer portion = \(12 \times 16 + 11 = 203\). Fractional portion = \(10/16 = 0.625\).
      Hence the answer is \((203.625)_{10}\).

   (b) (6 points) \((352.3)_{8} = (M)_{2} = (N)_{16}\).
      \textbf{Answer:}
      \[
      M = \hspace{2cm} N = \hspace{2cm}
      \]
      \((352.3)_{8} = (011 101 010.011)_{2} = (EA.6)_{16}\).

   (c) (5 points) A 12-digit \textit{Hexadecimal} number contains 9 integer digits and 3 fractional digits. Find the maximum \textit{decimal number} it can represent and the minimum non-zero \textit{decimal number} it can represent. You may represent the result as function of power of 2.
      \textbf{Answer:}
      \[
      \text{Maximum decimal number that can be represented} = \hspace{2cm}
      \]
      \[
      \text{Minimum non-zero decimal number that can be represented} = \hspace{2cm}
      \]
      \textbf{Max. decimal number:} The maximum \textit{whole} number in decimal is \((F \times 16^8 + F \times 16^7 + F \times 16^6 + F \times 16^5 + F \times 16^4 + F \times 16^3 + F \times 16^2 + F \times 16^1 + F \times 16^0) = (16^9 - 1)\) and the maximum possible fractional part in decimal is \((F \times 16^{-1} + F \times 16^{-2} + F \times 16^{-3}) = (16^{-3} - 1)/16^3\). So the maximum possible decimal number is obtained by adding whole number and fractional part:
      \[
      \text{Answer} = (16^9 - 1) + (16^{-3} - 1)/16^3
      \]
      \textbf{Min. non-zero decimal number} = \(16^{-3} = 2^{-12}\).

   (d) (5 points) A certain machine contains six different states, labeled symbolically by A, B, C, D, E, and F. Normally, the state transition cycles through a sequence A \(\rightarrow\) B \(\rightarrow\) C \(\rightarrow\) D \(\rightarrow\) E \(\rightarrow\) F \(\rightarrow\) A. Suppose that these states are to be encoded with 3-bit wide Gray code. Let A = 010. Find a valid 3-bit Gray code for the remaining states:
2. (20 points) arithmetic operations
   (a) (10 points) Find the radix \( r \) that satisfies the following equation
      \[
      (35)_{r} + (46)_{r+1} = (134)_{r+3}
      \]
      
      Answer: \( r = \) __________

      \[
      (3r + 5) + 4(r+1) + 6 = (r−3)^2 + 3(r−3) + 4, \ r > 5, r+1 > 6, \text{ and } r−3 > 4. \ \text{Simplify, we have}\ \]
      \[
      7r + 15 = r^2 − 6r + 9 + 3r − 9 + 4, \text{ or } r^2 − 10r − 11 = 0. \ \rightarrow (r−11)(r+1) = 0. \ \text{Thus, } r = 11 \text{ is the answer since it is required that } r > 7.\]

   (b) (10 points) Perform arithmetic operations in the following number representation.
      Indicate carries (for addition), borrows (for subtraction), and partial products (for multiplication) in addition to final answer.
3. (20 points) Truth table, canonical forms
   (a) (5 points) Represent the following Boolean function in sum of Minterms canonical form.
      \[ f(a,b,c) = a \cdot b + a \cdot \bar{c} + \bar{b} \cdot c \]
      \[ f(a,b,c) = ? \ m(1, 4, 5, 6, 7) \]
      Answer: \[ f(a,b,c) = \sum_m ( \ ) \).
   (b) (10 points) \( f(A,B,C) = (A + \bar{B})(B + \bar{C})(C + \bar{A}) \).
      Represent \( f(A,B,C) \) in product of Maxterms canonical form.
      Answer: \[ f(a,b,c) = \prod M( \ ) \]
      Note that \[ f(A,B,C) = \bar{A}B + \bar{B}C + \bar{C}A = \sum m(1,2,3,4,5,6) \]. Hence
      \[ f(A,B,C) = \prod M(1,2,3,4,5,6) \].
   (c) (5 points) Find the truth table of the Boolean function
      \[ f(w,x,y) = x \cdot w + \bar{x} \cdot \bar{w} + y(w + \bar{x}) \]
      \[
      \begin{array}{ccc|c}
      w & x & y & f(w,x,y) \\
      \hline
      0 & 0 & 0 & 1 \\
      0 & 0 & 1 & 1 \\
      0 & 1 & 0 & 0 \\
      0 & 1 & 1 & 0 \\
      1 & 0 & 0 & 0 \\
      1 & 0 & 1 & 1 \\
      1 & 1 & 0 & 1 \\
      1 & 1 & 1 & 1 \\
      \end{array}
      \]

4. (20 points) Boolean function realizations, K-Map
   (a) (10 points) Implement the following Boolean function using AND, OR, NOT logic gates.
      Do NOT assume the complement of a Boolean variable is available.
f(W,X,Y) = W \cdot \overline{X} + \left(\overline{W} \cdot (X + \overline{Y})\right)

(b) (5 points) Let \( f(a,b,c) = \overline{a} \cdot b \cdot \overline{c} + \overline{b} (a \cdot \overline{c} + \overline{a} \cdot c) \). Draw the K-map corresponding to \( f \).

Answer:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>00</th>
<th>01</th>
<th>11</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

(c) (5 points) The K-map of an unknown Boolean function \( f(a,b,c) \) is as follows:

<table>
<thead>
<tr>
<th>a \ b</th>
<th>c</th>
<th>00</th>
<th>01</th>
<th>11</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Simplify this Boolean function and give the result in sum of product (SOP) standard form.

Answer: \( f(a,b,c) = a \cdot b + b \cdot c + c \cdot a \).

5. (20 points) **Boolean Algebra, Simplification**

(a) (10 points) Find the complement of the following Boolean function and simplify the result in product of sum standard form.

\[
\begin{align*}
f(a,b,c) &= (a + b' + c)(a \cdot b + c) + a \cdot b \cdot c \\
f(a,b,c)' &= c'
\end{align*}
\]

Answer: \( f(a,b,c) = (a+b'+c)(a'b+c) + a'b'c = c \) since \( (a+b'+c)(a'b+c)=c \). \( f(a,b,c)' = c' \) Notice that \( f' \) is in both POS and SOP form since it is a function expressed with only a single literal.

(b) (5 points) Simplify the following Boolean function and represent the result in product of sum (POS) standard form.

\[
f(a,b,c) = \overline{M}(2,4,6)
\]

Answer: \( f(a,b,c) = \sum m(2,4,6) = a' \cdot \overline{c} + b \cdot c \). Hence \( f(a,b,c) = (a' + c)(b + c) \).

(c) (5 points) Suppose that \( x + y = 1 \). Proof the following identity algebraically.

\[
x \cdot \overline{y} + \overline{x} \cdot y = x + y
\]

(Hint: Use one or more Boolean identities listed in the last page of this exam.)

Answer:

| \begin{align*}
x \cdot \overline{y} + \overline{x} \cdot y \\
= (x + y)(\overline{x} + \overline{y}) \\
= 1 \cdot (\overline{x} + \overline{y})
\end{align*} | (x + y)(\overline{x} + \overline{y}) = x\overline{y} + \overline{x}y | x + y = 1 |
\[ x + y = x \]

\[ 1 \cdot x = x \]
### Basic Identities of Boolean Algebra

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<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>$X + 0 = X$</td>
</tr>
<tr>
<td>2.</td>
<td>$X \cdot 1 = X$</td>
</tr>
<tr>
<td>3.</td>
<td>$X + 1 = 1$</td>
</tr>
<tr>
<td>4.</td>
<td>$X \cdot 0 = 0$</td>
</tr>
<tr>
<td>5.</td>
<td>$X + X = X$</td>
</tr>
<tr>
<td>6.</td>
<td>$X \cdot X = X$</td>
</tr>
<tr>
<td>7.</td>
<td>$X + \overline{X} = 1$</td>
</tr>
<tr>
<td>8.</td>
<td>$X \cdot \overline{X} = 0$</td>
</tr>
<tr>
<td>9.</td>
<td>$\overline{X} = X$</td>
</tr>
<tr>
<td>10.</td>
<td>$X + Y = Y + X$</td>
</tr>
<tr>
<td>11.</td>
<td>$X \cdot Y = Y \cdot X$</td>
</tr>
<tr>
<td>12.</td>
<td>$X + (Y + Z) = (X + Y) + Z$</td>
</tr>
<tr>
<td>13.</td>
<td>$X (YZ) = (XY) Z$</td>
</tr>
<tr>
<td>14.</td>
<td>$X(Y + Z) = XY + XZ$</td>
</tr>
<tr>
<td>15.</td>
<td>$X + YZ = (X + Y)(X + Z)$</td>
</tr>
<tr>
<td>16.</td>
<td>$\overline{X + Y} = \overline{X} \cdot \overline{Y}$</td>
</tr>
<tr>
<td>17.</td>
<td>$\overline{X \cdot Y} = \overline{X} + \overline{Y}$</td>
</tr>
</tbody>
</table>

### Useful Boolean Identities

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$X + XY = X$</td>
<td>$X + \overline{XY} = X + Y$</td>
</tr>
<tr>
<td>$XY + X\overline{Y} = X$</td>
<td>$XY + \overline{XZ} + YZ = XY + \overline{XZ}$</td>
</tr>
<tr>
<td>$(X + Y)(\overline{X} + \overline{Y}) = X \cdot \overline{Y} + \overline{X} \cdot Y$</td>
<td>$(X + Y)(X + \overline{Y}) = X$</td>
</tr>
</tbody>
</table>