During the lecture, we haven’t really finished up the example on less than & equivalent relations of 4 variables K-map. Here is an example we partly covered in class with detailed explanation.

Problem: Given \( \{0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15\} \) find all prime implicants, essential prime implicants and find a sum of product expression with minimum literals.

Solution: First we find all prime implicants (rectangles containing largest number of 1’s in power of 2, say one 1, two 1s, four 1s, eight 1s, …). We have total of six prime implicants: \( \text{B'D}', \text{CD}, \text{B'C}, \text{BD}, \text{AB'}, \text{AD} \) as shown below.

An essential prime implicant is the only prime implicant that covers a minterm or minterms. In our problem, Minterm \( m_0 \) \( (=\text{A'B'C'D'}) \) is covered by only one prime implicants B’D’. Thus B’D’ is one of the essential prime implicants. Also Minterm \( m_5 \) \( (=\text{AB'C'D}) \) is only covered by the prime implicant BD, which is our second essential. We have total two essential prime implicants: B’D’ and BD as shown below. Notice covered minterms by essential prime implicants are now shown in gray and uncoverd minterms in black.
Now, we need to figure out how to cover uncovered minterms $m_3$, $m_9$, and $m_{11}$. First, let us focus on finding covers for minterms $m_9$ and $m_{11}$ (or a product term $AB'D$).

We have two prime implicants that cover both minterms $m_9$ and $m_{11}$, namely $AB'$ and $AD$. Naturally, we need to figure out which one would be better choice by establishing less than relation between $AB'$ and $AD$. The main idea is we want to identify less than prime implicant which we can remove from our consideration. Less than suggests it will contribute the less than the other (larger than) choice.
PI \_i \text{ is said to be } Less \ Than \ PI \_j \text{ if PI} \_i \text{ contains at least as many literals as PI} \_j \text{ and PI} \_j \text{ covers at least all of the as yet } uncovered \text{ minterms that PI} \_i \text{ covers. In other words, PI} \_i=\text{PI} \_j \text{ if 1) Number of literals: PI} \_i=\text{PI} \_j \text{ and 2) Coverage of uncovered minterms: PI} \_i=\text{PI} \_j \text{ (this is really a subset relation which is PI} \_i \text{ covers only the subset of uncovered minterms that PI} \_j \text{ covers.)}

If PI} \_i=\text{PI} \_j \text{ and PI} \_i=\text{PI} \_j \text{, then PI} \_i \text{ and PI} \_j \text{ are said to be equivalent which implies they have equivalent coverage so we can pick one.}

Let AB’=PI} \_i \text{ and AD}=PI} \_j \text{, as you can see from the figure, AB’ and AD has two literals each so we have satisfied condition 1) of less than relation. AB’ and AD also covers two uncovered minterms (shown in black), namely m\text{9} and m\text{11}. So we can say AB’=AD. However if we let AD=PI} \_i \text{ and AB’}=PI} \_j \text{, AD is also less than AB (AD=AB’) since they both have same number of literals (two of each), and they both cover identical uncovered minterms. Therefore, we can conclude that AB’ and AD are equivalent and we can make an arbitrary choice between them. We randomly choose AB’.

We now have to find a prime implicant which can cover remaining uncovered minterm, m\text{3}.

We now can see that there are two primary implicants which cover m\text{3}, namely B’C and CD. They both have 2 literals and cover same uncovered minterm m\text{3}, thus we have B’C=CD \text{ and B’C}=CD. So we conclude that they are equivalent and arbitrarily choose one, say CD.

Finally, we have covered all minterms and our minimum SOP expression is:
F= B’D’+CD+AB’+BD