Truth Tables

- Truth tables list the output of a function for all possible inputs:

<table>
<thead>
<tr>
<th>AND</th>
<th>OR</th>
<th>NOT</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>z = x, y</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>0</td>
<td>1</td>
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Hardware Example 1 – Switch Networks

For inputs, “logic 1” is switch closed, and “logic 0” is switch open.
For outputs, logic “1” is path closed and “logic 0” is path open.
For light, logic “1” is on and logic “0” is off.

Switches in parallel => OR

Switches in series => AND

Hardware Example 2 – Logic Gate

For $A = 1$ or $B = 1$, a path is closed to Fixed 0, so $X = 0$.
With $X = 0$, the path to Fixed 1 is closed, so that $F = 1$.

For $A = 0$ and $B = 0$, a path is closed to Fixed 1, so $X = 1$.
With $X = 1$, the path from Fixed 0 to $F$ is closed, so that $F = 0$.

What is the logic function?
Logic Gate Symbols

- **Logic Gates** are physical devices that implement binary logical functions.
- Logic gates are drawn as below and have a behavior with time as shown:

  \[
  F(X, Y) = X \lor Y
  \]

  \[
  F(X, Y) = X + Y
  \]

  \[
  F(X) = \overline{X}
  \]

Logic Diagrams and Expressions

Truth Table

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>Z</th>
<th>F = X + \overline{Y} \cdot Z</th>
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</thead>
<tbody>
<tr>
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Expression:

\[
F = X + \overline{Y} \cdot Z
\]

Boolean expressions, truth tables, and logic diagrams

**DESCRIBE THE SAME FUNCTION!**
Boolean Algebra

- An algebraic structure defined on a set of at least two elements, B together with two binary operators (denoted + and ) that satisfies the following identities:
  - Closure with respect to +, on B.
  - Identity elements 0 and 1 such that:
    1. \( X + 0 = X \)
    2. \( X + 1 = X \)
    3. \( X + 1 = 1 \)
    4. \( X + 0 = 0 \)
    5. \( X + X = X \)
    6. \( X + X = X \)
  - For each element X, an element X such that:
    7. \( X + X = 1 \)
    8. \( X + X = 0 \)

Boolean Algebra (Continued)

- The identities above are organized into dual pairs. These pairs have names as follows:
  1-4 Existence of 0 and 1
  5-6 Idempotence
  7-8 Existence of complement
  9 Involution
  10-11 Commuteive Laws
  12-13 Associativity
  14-15 Distributive Laws
  16-17 DeMorgan’s Laws

- We leave out the symbol if the meaning is unambiguous.
Properties of Identities & Algebra

- The dual of an algebraic expression is obtained by interchanging + and and interchanging 0's and 1's.
- The identities appear in dual pairs. When there is only one identity on a line the identity is self-dual, i.e., the dual expression = the original expression.
- There can be more that 2 elements in B, i.e., elements other than 1 and 0. What are some common useful Boolean algebras with more than 2 elements?
  - 1.
  - 2.
- If B contains only 1 and 0, then B is called the switching algebra which is the algebra we usually use in this course.

Boolean Algebraic Proofs - Example 1

- \( X + XY = X \)
Boolean Algebraic Proofs - Example 2

- $XY + XZ + YZ = XY + XZ$

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