Systematic Simplification Example

- Find the least literal cost solution considering both SOP and POS for $F(W, X, Y, Z) = \Sigma m(1, 5, 7, 8, 13, 15) + \Sigma d(2, 6, 9, 10)$

Results - Step 2

<table>
<thead>
<tr>
<th>Column (a)</th>
<th>Column (b)</th>
<th>Column (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>xyz</td>
<td>xyz</td>
<td>xyz</td>
</tr>
<tr>
<td>010 $m_0$</td>
<td>(01 - ) $m_0$</td>
<td>$m_0$</td>
</tr>
<tr>
<td>011 $m_1$</td>
<td>(-10) $m_0$, $m_1$</td>
<td>$m_1$</td>
</tr>
<tr>
<td>110 $m_2$</td>
<td>(-11) $m_0$, $m_2$</td>
<td>$m_2$</td>
</tr>
<tr>
<td>111 $m_3$</td>
<td>(11 - ) $m_0$, $m_3$</td>
<td>$m_3$</td>
</tr>
</tbody>
</table>

- Note that there are terms which are duplicated, - that is, they appear more than once.
- Duplicate terms:
  - have the same pattern of bits (including the -), and
  - came from the same minterms.
- Delete all but one of each duplicate term.

Tabular Method for PI Generation

- Alternative method for simplifying a Boolean equation
- Improved by Quine and McCluskey and known as the Q-M Method.
- The Tabular Method:
  1. Starts with a table of minterms.
  2. Compares each minterm with every other minterm in the list to find minterms which differ in exactly one variable.
  3. Constructs a new list with terms of one fewer variables, keeping track of which minterms were covered.
  4. Compares every element in the new list with each other to find terms that differ in one more variable.
  5. Repeats Steps 3 and 4 until done.

Step 3

<table>
<thead>
<tr>
<th>Column (a)</th>
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</thead>
<tbody>
<tr>
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<td>$m_2$</td>
</tr>
<tr>
<td>111 $m_3$</td>
<td>(11 - ) $m_0$, $m_3$</td>
<td>$m_3$</td>
</tr>
</tbody>
</table>

- Step 3: Make a new list by comparing items in Column (b)
  - 01: (-10 ⇒ Ø), (-11 ⇒ Ø), (11 ⇒ -1-)
  - -10: (01 ⇒ Ø), (-11 ⇒ -1-), (11 ⇒ Ø)
  - -11: (01 ⇒ -1-), (-10 ⇒ 1-1), (11 ⇒ Ø)
  - 11: (01 ⇒ -1-), (-10 ⇒ Ø), (-11 ⇒ Ø)
**Results - Step 3**

<table>
<thead>
<tr>
<th>Column (a)</th>
<th>Column (b)</th>
<th>Column (c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>xyz</td>
<td>xyz</td>
<td>xyz</td>
</tr>
<tr>
<td>010 m₁</td>
<td>(01-) m₂m₃</td>
<td>(-1-) m₂m₃m₆m₇</td>
</tr>
<tr>
<td>011 m₁</td>
<td>(-10) m₂m₃</td>
<td>(-1-) m₂m₆m₇</td>
</tr>
<tr>
<td>110 m₆</td>
<td>(-11) m₆m₇</td>
<td></td>
</tr>
<tr>
<td>111 m₇</td>
<td>(11-) m₆m₇</td>
<td></td>
</tr>
</tbody>
</table>

- Obviously, the Column (c) can be simplified by eliminating duplicates -- this leads to only one entry: (-1-) from m₂,m₃,m₆,m₇
- This corresponds to the Prime Implicant "y". The algorithm terminates at this point. Since this is the only PI and it covers all minterms, the result is: \( F(x,y,z) = y \)

**Computational Complexity Issues**

- The table method is complex. For "\( n \)" minterms, there are on the order of \( n^2 \) comparisons required.
- The Q-M Method simplifies the work by sorting the minterms into terms that can compare favorably. Terms that have no chance of combining are not even tried. It also adds some bookkeeping to simplify PI identification.
- Grouping: Use the number of 1s in the minterm to group the minterms. Preserve groups derived from this grouping in adjacent columns.
- Bookkeeping: Use a check mark (\( \checkmark \)) next to terms that have been combined.

**Q-M on \( F(x,y,z) = \Sigma m(2,3,6,7) \)**

Initial Group

<table>
<thead>
<tr>
<th>Column (a)</th>
<th>Column (b)</th>
<th>Column(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>xyz</td>
<td>xyz</td>
<td>xyz</td>
</tr>
<tr>
<td>010 m₂</td>
<td>(01-) m₂m₃</td>
<td>(-10) m₂m₆</td>
</tr>
<tr>
<td>011 m₃</td>
<td>(-10) m₂m₆</td>
<td>(-1-) m₂m₆m₇</td>
</tr>
<tr>
<td>110 m₆</td>
<td>(-11) m₆m₇</td>
<td></td>
</tr>
<tr>
<td>111 m₇</td>
<td>(11-) m₆m₇</td>
<td></td>
</tr>
</tbody>
</table>

- Step 2: Compare terms from adjacent groups.
  - Group 1 \( \Rightarrow \) Group 2
  - 010: (011 \( \Rightarrow \) 01-), (110 \( \Rightarrow \) -10)
  - Group 2 \( \Rightarrow \) Group 3
  - 011: (111 \( \Rightarrow \) -11)
  - 110: (111 \( \Rightarrow \) 11-)

- Step 4: Repeat on Column (b)
  - Group (1-2) \( \Rightarrow \) Group (2-3)
  - (01-): (-11 \( \Rightarrow \) Ø), (11- \( \Rightarrow \) -1-)
  - (-10): (-11 \( \Rightarrow \) -1-), (11- \( \Rightarrow \) Ø)

**Tabular Method: Cover Selection**

1. Construct a table with:
   a) Columns for each minterm, and
   b) Rows for each Prime Implicant.
2. Select Essential Prime Implicants and check off each covered minterm.
3. Delete Less Than Prime Implicants.
4. Select Secondary Essential Prime Implicants and check off each covered minterm.
5. Repeat 3 and 4 until a cover is generated.
6. If cycles exist, pick a PI and generate a cover and then delete that same PI and generate an alternate cover.
7. Select the minimum literal cover.
### Tabular Method Cover Example

**Function** \( g(w,x,y,z) \):

1. **Step 1:** Enter table:
   - | Type | 1-10 | 111- | 1-1 | 001- | 0-01 |
   - |     |      |      |     |      |
   - | -00 | x     | x     | x   |       |
   - | 00- | x     | x     | x   |       |
   - | 0-01 | x     |       |     |       |
   - | -101 | x     |       |     |       |
   - | 1-11 |       | x     | x   |       |
   - | 111- |       |       | x   | x     |
   - | 1-10 |       |       | x   | x     |

2. **Secondary Essential PIs**
   - Select secondary essential PIs and check them off along with minterms covered.
   - | Type | 1-10 | 111- | 1-1 | 001- |
   - |     |      |      |     |      |
   - | -00 |       |       |     |       |
   - | 00- |       |       |     |       |
   - | 0-01 |       |       |     |       |
   - | -101 |       |       |     |       |
   - | 1-11 |       |       |     |       |
   - | 111- |       |       |     |       |

3. **Less Than Prime Implicants**
   - 0-01 \( \leq \) 1-10
   - 1-10 \( \leq \) 111-

4. **Cyclic Structures**
   - Let \( F(x,y,z) = \Sigma m(0,1,2,5,6,7) \)
   - Enter table:
     - | Type | 00- | 01- | 10- |
     - |     |     |     |     |
     - | -00 | x   |     |     |
     - | 00- | x   |     |     |
     - | 01- | x   |     |     |
     - | 10- | x   |     |     |
     - | 11- | x   |     |     |
     - | 110-| x   |     |     |

**Results for** \( g \)

- Note that after the secondary essentials have been added, PI 11-1 \((w \land x \land z)\) is not needed to cover minterms, so it is **redundant**.
- \( F(w,x,y,z) = x \land z + w \land x \land y \land z + w \land x \land y \)
### Cyclic Structure: Pick PI

**Step 1:** Pick a PI and mark off the minterms covered.

<table>
<thead>
<tr>
<th>Pls</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Picked)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 00-  | x | x |
- 01-  | x | x |
- 1-1  | x | x |
- 11-  | x | x |
- -10  | x | x |
- 0-0  | x | x |

**Step 2:** Eliminate less than PIs.

<table>
<thead>
<tr>
<th>Pls</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Picked)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 00-  | x | x |
- 01-  | x | x |
- 1-1  | x | x |
- 11-  | x | x |
- -10  | x | x |
- 0-0  | x | x |

### Start Over: Deleting Selected PI

**Step 1:** Delete the PI picked.

**Step 2:** Select essential PIs.

**Step 3:** Find and delete less than PIs.

<table>
<thead>
<tr>
<th>Pls</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td>x</td>
<td></td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Picked)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 00-  | x | x | (Deleted) |
- 01-  | x | x | Essential |
- 1-1  | x | x | Essential |
- 11-  | x | x | Essential |
- -10  | x | x |
- 0-0  | x | x | Essential |

**Result:** \( F(x, y, z) = \overline{y}z + xy + \overline{x}z \)

**Both results are minimum literal. Use either.**

### An Example with Don’t Cares

**F(A, B, C, D) = \sum(2, 4, 5, 13, 14, 15) + \sum(0, 1, 6, 10)**

**Using K-Map to get PIs:**

- \( F(x, y, z) = xy + xz + yz \)
An Example with Don’t Cares

- Using Tabular Selection:

<table>
<thead>
<tr>
<th>PIs</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Result:

Minimum POS Example

- Given \( g(w,x,y,z) \), find PIs for 0s.

\[
\begin{array}{c}
\begin{array}{cccc}
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
\end{array}
\end{array}
\]

- Selecting PIs: \( g = XYZ + X'Y + YZ' \)
- Complementing: \( g' = (x + y + z)(w + x + y)(w + x + z) \)

Minimum POS

- We can use the minimization techniques learned so far to implement a minimum literal, standard POS form:
  - Simplify \( F \) using SOP methods.
  - Complement the result for \( F \).
  - On a K-map, note that \( F \) corresponds to the 0s.