1. (20 points) number representations and conversion

(a) (5 points) \((63)_{10} = (\text{ })_7\)

**Solution:** We repeatedly divide 63 by 7 and write the remainder in reverse order.

\[
\begin{align*}
63/7 &= 9 + 0/7 \\
9/7 &= 1 + 2/7 \\
1/7 &= 0 + 1/7
\end{align*}
\]

\((63)_{10} = (120)_7\)

(b) (5 points) Determine the radix \(r\) for following case.

\((246)_r = (132)_{10}\)

**Solution:** If we expand the left hand side equation, we get

\[
\begin{align*}
2r^2 + 4r + 6 &= 132 \\
r^2 + 2r + 3 &= 66 \\
r^2 + 2r - 63 &= 0
\end{align*}
\]

\((r + 9) (r - 7) = 0; \text{ since } r > 0, \text{ we have } r = 7.\)

(c) (5 points) Find the Octal representation of the following Hexadecimal number:

\((8A3E.1)_{16} = (\text{ })_8\)

**Solution:** First, find the representation in base 2, then group three bits as one octal digit.

\[
\begin{align*}
(8A3E.1)_{16} &= (1000\ 1010\ 0011\ 1110\ .\ 0001)_{2} \\
&= (001\ 000\ 101\ 000\ 111\ 110\ .\ 000\ 100)_{2} \\
&= (105076\ .\ 04)_8
\end{align*}
\]

(d) (5 points) Find the binary representation for the BCD number

\(1001\ 0000\ 1000_{\text{BCD}}\)

**Solution:** 1001 0000 1000\(_{\text{BCD}}\) equals to 908\(_{10}\). If you divide 908\(_{10}\) repeatedly with 2, we get 1110001100\(_2\) as its binary representation.
2. (20 points) arithmetic operations, binary code
   (a) (10 points) Perform arithmetic operations in the following number representation. 
   Indicate carries (for addition) and borrows (for subtraction) in addition to final answer.

   (5 points)  
   Binary  
   Carries 0 1 1 1 0  
   + 0 0 1 1 1  
   1 0 1 0 1  

   (5 points)  
   Binary  
   Borrows 0 1 0 1 1  
   − 0 1 0 1 1  

   (b) (10 points) Perform the following BCD addition arithmetic operations in the space provided below. You must show all your work to receive full credit.

   \[ 387_{10} + 439_{10} \]

   Solution: \[ 0011\ 1000\ 0111_{BCD} + 0100\ 0011\ 1001_{BCD} = 1000\ 0010\ 0110_{BCD} \]

3. (20 points) Boolean Algebra, Truth table, canonical forms
   (a) (10 points) Express the following Boolean function in minimized product of sum form. 
   You must show all your work to receive full credit.

   \[ F(X,Y,V,W) = X \cdot \overline{Y} \cdot \overline{W} + X \cdot Y \cdot W + \overline{X} \cdot Y \cdot V + \overline{X} \cdot V \cdot \overline{W} \]

   Solution: This problem should be solved in two steps. First, we need to find \( F(X,Y,V,W) \) in S.O.P (sum of product) form. Using 4-variable K-map, we can write 1’s in minterms covered by function \( F(X,Y,V,W) \), namely \( X \cdot \overline{Y} \cdot \overline{W} + X \cdot Y \cdot W + \overline{X} \cdot Y \cdot V + \overline{X} \cdot V \cdot \overline{W} \). Then we find \( F(X,Y,V,W) = \overline{X} \cdot \overline{Y} + \overline{Y} \cdot \overline{W} + X \cdot Y \cdot \overline{W} \) by covering 0’s in the K-map. Secondly, we obtain \( F(X,Y,V,W) = (X + V) \cdot (Y + \overline{W}) \cdot (\overline{X} + \overline{Y} + W) \) in P.O.S. by finding a dual of \( F(X,Y,V,W) \) and complementing each literal.
(b) (10 points) Express the following Boolean function in a minimized sum of product form. You must show all your work to receive full credit.

\[ F(A, B, C, D) = \overline{M}(1, 3, 6, 13, 14, 15). \]

**Solution:**
\[
F(A, B, C, D) = \overline{B} \cdot \overline{D} + \overline{C} \cdot \overline{D} + \overline{A} \cdot B \cdot D + A \cdot \overline{B}
\]

Since the function \( F(A, B, C, D) \) is given in Product of Maxterms, we need to find the function \( F(A, B, C, D) \) in Sum of Minterms, which is \( F(A, B, C, D) = \sum m(0, 2, 4, 5, 7, 8, 9, 10, 11, 12) \). Then, we simply find the minimized sum of product using a 4-variable K-map.

4. (10 points) Boolean Algebra
Prove the following identity, \( F(W, X, Y, Z) \), algebraically.

\[
W \cdot \overline{X} + \overline{W} \cdot \overline{Y} \cdot Z + \overline{W} \cdot \overline{X} \cdot Z + \overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot \overline{Z} = \overline{X} + \overline{W} \cdot \overline{Y} \cdot \overline{Z}
\]

There is no need to explicitly list the use of commutative law as it is used frequently. You may use all the Boolean identities listed at the end of this exam paper (page 10). You should not need more than the spaces provided.

<table>
<thead>
<tr>
<th>Boolean Expression</th>
<th>Boolean Identity used</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W \cdot \overline{X} + \overline{W} \cdot \overline{Y} \cdot Z + \overline{W} \cdot \overline{X} \cdot Z + \overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot \overline{Z} )</td>
<td>2, 3, 7</td>
</tr>
<tr>
<td>( = W \cdot \overline{X} + \overline{W} \cdot \overline{Y} \cdot Z \cdot (\overline{X} + 1) + \overline{W} \cdot \overline{X} \cdot Z \cdot (\overline{Y} + Y) + \overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot \overline{Z} )</td>
<td>14</td>
</tr>
<tr>
<td>( = W \cdot \overline{X} + \overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot Z + \overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot Z + \overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot \overline{Z} + \overline{W} \cdot \overline{Y} \cdot \overline{Z} )</td>
<td>20</td>
</tr>
<tr>
<td>( = W \cdot \overline{X} + \overline{W} \cdot \overline{X} \cdot \overline{Y} + \overline{W} \cdot \overline{X} \cdot \overline{Y} + \overline{W} \cdot \overline{X} \cdot \overline{Y} \cdot \overline{Z} )</td>
<td>20</td>
</tr>
<tr>
<td>( = W \cdot \overline{X} + \overline{W} \cdot \overline{X} + \overline{W} \cdot \overline{Y} \cdot \overline{Z} )</td>
<td>20</td>
</tr>
<tr>
<td>( = \overline{X} + \overline{W} \cdot \overline{Y} \cdot \overline{Z} )</td>
<td>20</td>
</tr>
</tbody>
</table>
5. (15 points) Systematic Boolean simplification
   (a) (10 points) Express the function \( F(A, B, C, D) = \overline{m}(1, 5, 6, 8, 10, 11, 13, 15) \) using a 
   minimized sum of product standard form. Find three different minimized sum of 
   product forms for function \( F(A, B, C, D) \) with the minimum literal cost. You must show 
   all your work to receive full credit.

   **Solution:** Using K-map, we find the three Essential Prime Implicants, which are 
   \( \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D \), \( A \cdot \overline{B} \cdot \overline{D} \), and \( \overline{A} \cdot B \cdot C \cdot \overline{D} \). There are still three uncovered minterms 
   remaining, namely \( m_{11}, m_{13} \) and \( m_{15} \). Now there are three ways to cover \( m_{11}, m_{13} \) and \( m_{15} \) 
   using two product terms each with exactly three literals: \( \{ B \cdot \overline{C} \cdot D + A \cdot C \cdot D \} \), 
   \( \{ A \cdot B \cdot D + A \cdot C \cdot D \} \), and \( \{ A \cdot B \cdot D + A \cdot \overline{B} \cdot C \} \). Thus, we have following three 
   solutions with exactly same literal count.

   \[
   F(A, B, C, D) = \overline{A} \cdot \overline{C} \cdot D + A \cdot \overline{B} \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot \overline{D} + \{ B \cdot \overline{C} \cdot D + A \cdot C \cdot D \} \\
   F(A, B, C, D) = \overline{A} \cdot \overline{C} \cdot D + A \cdot \overline{B} \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot \overline{D} + \{ A \cdot B \cdot D + A \cdot C \cdot D \} \\
   F(A, B, C, D) = \overline{A} \cdot \overline{C} \cdot D + A \cdot \overline{B} \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot \overline{D} + \{ A \cdot B \cdot D + A \cdot \overline{B} \cdot C \}
   \]

   (b) (5 points) Suppose the complements of inputs are not available and inputs are needed to 
   be complemented for the use of each product term. Thus, use of the sum of product form 
   with a minimum number of complemented literals is desired. Which sum of product term 
   would you use among three possible sum of product forms you have found from part (a)? 
   Using only a sentence or two briefly explain your reasoning.

   **Solution:** \( F(A, B, C, D) = \overline{A} \cdot \overline{C} \cdot D + A \cdot \overline{B} \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot \overline{D} + \{ A \cdot B \cdot D + A \cdot C \cdot D \} \) is 
   preferred since there are total of 6 complements, whereas other two require total of 7 
   complements.

6. (15 points) Systematic Boolean simplification
   (a) (7 points) Consider the Boolean function below:

   \( F(a, b, c, d) = \overline{M}(1, 4, 5, 6, 9, 10) \).

   Find ALL the prime implicants of function \( F(a, b, c, d) \) using the tabular method. Answers 
   without work will not receive any credit!

   **Solution:** Currently there are more than one notation being used for the tabular method. 
   Following is a solution using a group notation \( G_x \), where \( x \) is the number of 1’s in each cube. 
   Alternatively, some notation may list all combined minterms next to each cube.

   PIs are all the unchecked cubes (those without the check marks next to them) with proper literals 
   in place of 0’s or 1’s. There are six unchecked cubes remaining, namely 00-0, -000, 001-, 1-00, - 
   -11 and 11-- (in cube notation), thus prime implicants are \( \overline{a} \cdot \overline{b} \cdot d \), \( b \cdot \overline{c} \cdot \overline{d} \), \( \overline{a} \cdot \overline{b} \cdot \overline{c} \), \( a \cdot \overline{c} \cdot \overline{d} \), 
   \( c \cdot d \), and \( a \cdot b \).
(b) (8 points) Boolean function \(g(w,x,y,z)\) consists of the following six prime implicants:

\[
\begin{align*}
& w \cdot x \cdot z, \quad w \cdot x \cdot y, \quad w \cdot y \cdot z, \quad w \cdot x \cdot z, \quad w \cdot x \cdot y, \quad y \cdot z
\end{align*}
\]

Use a covering table, categorize these PIs into (i) Essential PI(s) (EPI), (ii) Less-than PI(s) (LTPI), (iii) Secondary Essential PI(s) (SEPI), or (iv) Redundant PI(s) (RPI).

**Solution:** Depending on the order of your less than PI elimination, you may get slightly different results. Following solution is the one you may obtain if you try to cover EPIs first, then proceeds with an uncovered minterm with smallest index first (starting from left to right columns). The numbers inside of parenthesis in PI category represent the order of categorization, and the numbers on the bottom of the table next to red vertical lines indicate when each column was crossed out.

<table>
<thead>
<tr>
<th>(w \times x \times y \times z)</th>
<th>(0000)</th>
<th>(0001)</th>
<th>(0010)</th>
<th>(0101)</th>
<th>(0111)</th>
<th>(0110)</th>
<th>(1010)</th>
<th>(1110)</th>
<th>(\text{PI category})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0 0 0 0)</td>
<td>(\text{X})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LTPI (2)</td>
</tr>
<tr>
<td>(0 0 0 1)</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SEPI (3)</td>
</tr>
<tr>
<td>(0 0 1 1)</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0 1 0 1)</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LTPI (4)</td>
</tr>
<tr>
<td>(0 1 1 0)</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SEPI (5)</td>
</tr>
<tr>
<td>(1 0 1 1)</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>RPI (6)</td>
</tr>
<tr>
<td>(1 1 1 1)</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td>(\text{X})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>EPI (1)</td>
</tr>
</tbody>
</table>
Basic Identities of Boolean Algebra

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$X + 0 = X$</td>
</tr>
<tr>
<td>2.</td>
<td>$X \cdot 1 = X$</td>
</tr>
<tr>
<td>3.</td>
<td>$X + 1 = 1$</td>
</tr>
<tr>
<td>4.</td>
<td>$X \cdot 0 = 0$</td>
</tr>
<tr>
<td>5.</td>
<td>$X \cdot X = X$</td>
</tr>
<tr>
<td>6.</td>
<td>$X + X = X$</td>
</tr>
<tr>
<td>7.</td>
<td>$X + \overline{X} = 1$</td>
</tr>
<tr>
<td>8.</td>
<td>$X \cdot \overline{X} = 0$</td>
</tr>
<tr>
<td>9.</td>
<td>$\overline{\overline{X}} = X$</td>
</tr>
<tr>
<td>10.</td>
<td>$X + Y = Y + X$</td>
</tr>
<tr>
<td>11.</td>
<td>$X \cdot Y = Y \cdot X$</td>
</tr>
<tr>
<td>12.</td>
<td>$X + (Y + Z) = (X + Y) + Z$</td>
</tr>
<tr>
<td>13.</td>
<td>$X (YZ) = (XY) Z$</td>
</tr>
<tr>
<td>14.</td>
<td>$X(Y + Z) = XY + XZ$</td>
</tr>
<tr>
<td>15.</td>
<td>$X + YZ = (X + Y)(X + Z)$</td>
</tr>
<tr>
<td>16.</td>
<td>$X + Y = \overline{X} \cdot \overline{Y}$</td>
</tr>
<tr>
<td>17.</td>
<td>$X \cdot Y = \overline{X} + \overline{Y}$</td>
</tr>
</tbody>
</table>

Useful Boolean Identities

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18.</td>
<td>$X + XY = X$</td>
</tr>
<tr>
<td>19.</td>
<td>$X + \overline{XY} = X + Y$</td>
</tr>
<tr>
<td>20.</td>
<td>$XY + X\overline{Y} = X$</td>
</tr>
<tr>
<td>21.</td>
<td>$XY + \overline{XZ} + YZ = XY + \overline{X} Z$</td>
</tr>
<tr>
<td>22.</td>
<td>$(X + Y)(X + \overline{Y}) = X \cdot \overline{Y} + \overline{X} \cdot Y$</td>
</tr>
<tr>
<td>23.</td>
<td>$(X + Y)(X + \overline{Y}) = X$</td>
</tr>
</tbody>
</table>