Magnetic Resonance Imaging:
Single Coil Sensitivity Mapping and Correction using
Spatial Harmonics

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**Motivation**

In the case of the hyperpolarized gas imaging of the lungs, a cost effective vest-coil (to be described in detail in the Hyperpolarized Gas section) is used. Normally this coil geometry gives nearly ideal coil sensitivity. However, not all patients are of similar sizes, and therefore when imaging larger patients, the vest coil does not completely circumscribe the patient. This leads to severe non-uniformities in the sensitivity profile near the open section of the vest coil, following the $1/r^2$ principle of coil sensitivity. This, in turn, leads to significant attenuation of the MR signal from the affected portion of the lung volume.

The goal of this project is to correct the inhomogeneities using only the acquired scan. This would allow complete backwards compatibility with all previous scans, and allow the process to work without requiring any additional use of the rather expensive $^3$He gas. It would be a success if the correction preserved the ventilation defects, as described in the Background section, and the ADC map, also described in the Background section, while simultaneously correcting for the coil inhomogeneities to some extent. The theory and methods sections, later in the paper, describe this process in far greater detail.

**Background**

**Basics of Magnetic Resonance Imaging**

Magnetic Resonance Imaging (MRI) is a type of medical imaging system which uses strong magnetic fields and RadioFrequency (RF) pulses and coils to produce images. The basic requirements for MRI are that the object being imaged can tolerate the high magnetic fields, does not produce a major magnetic field itself, and that it is or contains non-zero spin molecules as defined by quantum mechanics. The basic principle behind the imaging is to excite the atoms of the object using an RF pulse, and then to ‘listen’ to the RF output as they decay back to a steady state. Using that basic principle, research and clinically relevant images can be acquired using non-ionizing radiation.

The RF energy used is at a frequency of $\gamma B$ where $\gamma$ is the gyromagnetic ratio of a specific atom, and B is the magnetic field strength. The typical imaging of water uses a frequency of ~64MHz because $\gamma$ is ~43MHz/T and B is typically 1.5T where T is the magnetic field strength in Tesla [9]. The MRI hardware has to be tuned very precisely to that frequency in order to maximize the signal and Signal to Noise Ratio (SNR).
The SNR for a typical MRI scan is largely based on the water concentration. However, if a constant, 100% water concentration is used, the SNR equation is

$$\text{SNR} = (\text{Voxel Volume}) \times \sqrt{\text{Data Acquisition Time}}$$ [9]. From this equation, it is obvious that larger voxel volumes and more data gives a better SNR, conditions that do not, however, always give the best images because of the multitude of other factors involved.

**Coil Sensitivity (theory and practice)**

In this paper we are most intimately concerned with exploiting the physics of MR signal acquisition. An entire sub-section of the medical MR imaging field is devoted to employing these vary techniques to accelerate the signal acquisition; and therefore the temporal resolution of the signal. This practice, called Parallel Magnetic Resonance Imaging (ll MRI), will be discussed later on. First, the physics of the situation should be described.

In MRI, the spins of the protons within the sample to be imaged (tissue, phantom, hyperpolarized gas, etc) reach an equilibrium with the latent magnetic field of the MR scanner \((B_0)\). The z-axis is defined in the direction of this magnetic field. Essentially the net magnetization vector \((M)\) of the sample is nearly parallel to, and precesses about, \(B_0\). The frequency with which \(M\) precesses about the z-axis is an inherent property of the atomic or molecular species containing the proton [8]. Greater details concerning this topic are beyond the scope of this discussion, but interested readers are referred to [9].

The reason that MR imaging works is that RF energy is driven into the sample, which causes \(M\) to be “flipped” into the xy plane. The precession of \(M\) in the xy plane will create a fluctuating magnetic field, which will induce a current in a loop of wire that is orthogonal to the xy plane. Such a coil is the most basic type of MR receive coil; the current is the MR signal. From basic electro-magnetism, it is well known that the magnitude of the fluctuating magnetic field at the coil is proportional to \(1/r^2\); where \(r\) is the distance from the precessing spin to the coil.

The main concern in MRI coil geometry design is to achieve a (nearly) uniform sensitivity to these precessing spins throughout a designated imaging volume. However, other concerns in design exist which may drive the coil design to have a less than ideal (uniform) sensitivity profile throughout the imaging volume. A sample of these concerns includes: very high temporal resolution (ll MRI), very high spatial resolution (single coils built into catheters), coils tuned to a less commonly imaged frequency of precession
(hyperpolarized gas imaging), or cost. Any combination of these concerns may lead to non-ideal coil sensitivity profiles.

**Parallel Imaging**

The most common type of non-ideal coil geometry used is encountered during fast imaging protocols that utilize multiple receive coils in parallel. In general any number of coils can be arrayed around an object volume and then used in parallel to image the entire volume. To demonstrate the concept, let us consider a circular, 2D object. If four coils are arranged around the object, each coil will be very sensitive to the portion of the object which is nearest to the coil, and much less sensitive to the portion of the object furthest away ($1/r^2$). Each individual coil is capable of creating an MR image of the object, just not a very good one. These individual images are then combined, using coil sensitivity maps, to produce an improved image.

**Hyperpolarized Gas Imaging**

Typically, MRI is used to image water in the body simply because it is plentiful and inherently has an excellent signal. However, other molecules can be imaged, and for this project images acquired using Hyperpolarized $^3$He (HP $^3$He) are used. Hyperpolarization is simply the process of increasing the magnetization beyond steady state. This is done because the inherent signal from $^3$He is very low under normal scanning conditions [6].

The reason for using HP $^3$He in this case is that the lungs inherently have very little water, which makes normal water images difficult to acquire. By using HP $^3$He, a functional lung image can be obtained, examples of which are shown in figure 2.

As stated above, the precession frequency is $\gamma B$, and $\gamma$ for $^3$He is approximately 20 MHz/T which is less than half that of water [6]. Because of the fine tuning required for the coil at excitation and receive, a normal, water tuned coil can not be used for HP $^3$He imaging. Also, because of the varying sizes of people, and the requirement of lung imaging, the coils are typically designed as vest coils, which can be formed to the person. This is good in some respects, but it gives a non-uniform sensitivity over the lung volume if the person is too large or too small for the coil. Varying coil sensitivity is an issue, because it can make parts of the lung look artificially dark or bright.

Because the coil is tuned to $^3$He, and the $^3$He is hyperpolarized, normal methods for acquiring coil sensitivity maps, such as phantoms and multiple images, are not
applicable. This is because, for every excitation, some of the hyperpolarization is used, so due to of the physics, it is difficult to image. Also, a person can only hold their breath for about 30 seconds on average, especially if they have a lung condition. Because of this, an image must be acquired with very little excitation, which implies very little signal received, and in only ~30 seconds; these are very severe constraints for a typical MRI system. Because of those constraints, the only information acquired is the image, and therefore any coil sensitivity information must be gleaned from the image itself.

**Quantitative and Qualitative Lung Measures**

Apparent Diffusion Coefficient (ADC) imaging is a powerful tool because it allows quantitative analysis of diffusion in any part of the image. This metric is used extensively in lung imaging because it can effectively measure the breakdown of the alveoli caused by various lung diseases by ‘seeing’ the increased diffusion present in those cases.

ADC is an important validation step because it is calculated on a point by point basis from two images, and if both images are not correct, large artifacts are easily induced in the ADC image because of the nature of the ADC equation as shown below.

\[
ADC = \log(S/So) \cdot b
\]

where \( So \) is the un-weighted image, \( S \) is the weighted image, and \( b \) is the diffusion weighting applied to the weighted image [6]. Because of this, the ADC map is an excellent metric of quality and reliability of the coil sensitivity correction.

Defect score is another measure of lung health. It is currently a qualitative score, but there are methods under development which will transform it into percent defected. Currently, however, defect score is a qualitative measure of the number and size of ventilation defects in the lung [6]. Ventilation defects are areas of the lung where there is little or no signal because the \(^3\)He does not reach that area. Several can be seen in figures 6 and 7, one of which is highlighted by the red box. Ventilation defects such as these are an important tool in regional lung analysis and as with ADC, should be preserved by any coil sensitivity correction.

**Theory**

A MR imaging and reconstruction method using multiple coils called Generalized SMASH (SiMultaneous Acquisition of Spatial Harmonics) was introduced in 2002[1]; which is itself an extension of SMASH[2]. Generalized SMASH proposed a method by
which coil sensitivity maps could be generated using linear combinations of spatial harmonics.

\[ C(x, y) = \sum_{m=-p}^{q} a^m(x)e^{-im\Delta ky} \]

Equation 1: Spatial Harmonic Equation [1]

Where \( p \) and \( q \) are integers determined to provide an adequate coil sensitivity representation, \( a^m \) is the \( m \)th complex Fourier coefficient. This algorithm consists of using few (small \( p \) and \( q \) from Equation 1) low frequency harmonics as an estimate of the coil sensitivity.

We propose a novel application of this method as an image restoration algorithm for non-ideal single coil geometries. In such non-ideal geometries, the MR image acquired is assumed to be a degraded version of the original (ideal) image where the degradation model is quite simple. The Nuclear Magnetic Resonance (NMR) signal \( (M(x,y)) \) is degraded by the non-uniform coil sensitivity profile over the imaging volume \( (C(x,y)) \) in the presence of zero-mean additive noise \( (\eta(x,y)) \):

\[ S(x, y) = M(x, y) \ast C(x, y) + \eta(x, y) \]

Equation 2: Image Degradation Model

In the absence of noise, solving equation 2 for \( M(x,y) \) reduces to a simple inversion problem (figure 1). That is, if you know the coil sensitivity map \( (C(x,y)) \) then you simply multiply \( S \) by \( C^{-1} \) and the problem is solved. However, we don’t know \( C \) and we can’t exactly know the noise distribution.
**Figure 1:** Graphical representation of the image degradation model in both image space and in Fourier space. Note the coil sensitivity amplitude (from a single coil) falls off as $1/r^2$. *figure adapted from Samsonov [7].

However, we know that all regions with a SNR below some threshold should not be amplified. These regions are below the noise floor. As per the standard in ADC calculations in MR diffusion imaging, the low amplitude additive noise will be thresholded out using a SNR threshold [5].

Equation 1 is used to calculate an approximated coil sensitivity map. From the calculated sensitivity map, the inverse can be computed and modulated to attenuate high amplification elements. The inverted and modulated sensitivity map will be multiplied by the original image to produce the “corrected” image.

There are some assumptions used in this algorithm that in almost all cases will not be fully met. Certainly they are not all met in the application of hyperpolarized gas MRI of the lungs. The main assumption in using SMASH, SENSE or GRAPPA techniques for parallel MR imaging (and therefore equation 1) is that at low frequencies, the NMR signal across the object volume can be well approximated as uniform in space. This is what allows the statement that if only the center of Fourier Space data is used and the rest set to zero, and then the 2D-DFT will result in spatial variance which derives from the coil sensitivity alone. To the extent that this assumption is not met, the calculated coil sensitivity map and resultant “corrected” image will only be approximations; and to some
extent dependent upon the number of harmonics used to calculate the approximated sensitivity map.

While accepting that any image restoration is to some extent artificial, or subjective due to the unknown elements of the non-degraded image, we propose to demonstrate the utility of this particular restoration algorithm. There are a number of post-processing analyses performed on hyperpolarized gas MRI of the lungs as regional metrics of various physiological states. Any image restoration algorithm will need to improve at least one analysis and degrade no analyses of the MR data to be accepted into routine practice. We will test this algorithm’s performance on two of the analyses most commonly utilized; ventilation defect detection and ADC mapping.

Ventilation defect detection is, in general, an observer dependent analysis. This is where we expect that image restoration has potential to increase the utility of this analysis. That is, if the attenuated data can be appropriately amplified, then the observer can detect the ventilation defects apparent in those regions of abnormal local variations within the lung volume. In contrast, ADC calculations depend on the log of the ratio of values from two images acquired under different conditions [4]. This is a user independent calculation that functions in the presence of the image degradation. Any image restoration algorithm must ensure that while the degradation is approximately removed, the ADC calculation is not significantly perturbed. Our methods for performing this calculation will be outlined later in this paper, the interested reader is also referenced Chen et al [4].
**Methods**

Table 1: Block diagram for the MATLAB code
Details

The first step of the implementation, the image input block is present to indicate the beginning of the process. The input can theoretically be in any image format, but in this case DICOM images were used.

The noise find block was implemented by specifying a fixed ROI outside the lung area. In this case, it was a region known to be outside the lung in every patient, so a fixed ROI was a reliable measure for finding the noise threshold.

The noise and signal images were then split by using a threshold determined to be 5 times the standard deviation of the noise. Every voxel above that value was determined to have a high enough SNR to be part of the region, and every voxel below the value was determined to be noise.

The crop step for both the noise and signal images was an optional step where the image was cropped to a smaller size by removing noise regions evenly on all sides. This was not often used, and did not change the results by an appreciable amount.

The FFT block was the simple Fast Fourier Transform of the image. This was done because equation 1 specifies that spatial harmonics are equivalent to the DFT of the image, which is the same as the FFT.

The next step was to select the number of harmonics to keep. Sizes between 3x3 and 11x11 were tried, with 7x7 being a reliably good number. This is an important step, because with too many harmonics, too many high frequencies are used to compute the coil sensitivity map, and unwanted signal gains are produced. With too few harmonics, there is not enough variability in the sensitivity map to correctly reproduce the coil sensitivity profile.

The IFT was simply the step that transforms the coil harmonics back into image space. The result of this step is the production of the coil sensitivity map.

The coil sensitivity map is then inverted using a simple inversion with a small additional factor to prevent division by zero. \( I = 1/(S + 0.01) \) The minimum of the inverted map was then set to 1 to prevent unwanted signal loss in any part of the image.

The very direct inversion results in very high values in some places, this is corrected by using a user defined triangular filter. In practice, all gains above 10 were attenuated back down to 1 with a slope of -1, and anything above 19 was set to 1 directly. Other methods for non-linearly reducing the gain, such as subtracting the log of the gain were attempted in addition to the triangular filter, however qualitatively there was little difference in the resulting image. The reason a triangular filter was used was because the
extremely high gains were in the noise field, so even though the noise was removed, some extremely high noise voxels still remained, therefore the less noise enhancement, the better.

At this point the corrected image can be found by simply doing a point by point multiplication of the inverted sensitivity map with the signal image. Theoretically the image could have been divided by the sensitivity map, but the current implementation makes the modifications of the inverse sensitivity map far easier.

The final signal modification step was to normalize the means of the original and corrected images. The mean is used because it is a reliable measure of signal modification, and other measures such as the maximum are not as reliable because of the high gain in some parts of the image.

The final step was to add the noise image and the signal image back together. This simply makes the image more natural to look at, and the noise threshold was already low enough so that it would not disturb further analysis of the image.

Results

Case 1: The patient is an asthma patient, representing typical lung function. However, due to the physical extent of the patient, vest coil would not completely wrap around the patient's chest. This leads to a very non-ideal coil geometry and resulted in very non-uniform intensities across the field of view (figure 2). This patient is included here as a representative of the extreme end of single coil non-uniform sensitivity artifact. This patient was identified as the 11th member of this study. Therefore, for the remainder of this paper, the handle “image set 11” will be assigned to this case's data. Note that the information contained in the left half of the image is nearly lost, and is impossible to extricate from the background. Figure 3 demonstrates the results of implementing this algorithm. Notice that the information in the left side of the field of view has been drawn out of the background. However, there are areas where the information was below the noise floor, and therefore unrecoverable.

Case 2: The patient is an asthma patient, representing typical lung function and image quality associated with this data set. The patient's lungs exhibit sporadic defects and a signal inhomogeneity common to people exhibiting asthma like symptoms. This patient is included here as a representative of the majority and common state of hyperpolarized $^3$He MR imaging of the lungs in both the clinical and medical research
fields. This patient was identified as the 18\textsuperscript{th} member of this study. Therefore, for the remainder of this paper, the handle “image set 18” will be assigned to this case's data. Figure 4 illustrates the original, degraded data for image set 18. Note the spatial variance across the field of view. Figure 5 shows the resulting image set after implementation of the sensitivity correction algorithm. Figure 7 illustrates the degree to which the results of the algorithm; showing a degraded image, a profile across one row, the corrected image and a profile across the same row. Notice the corrected image displays a much more uniform profile across the field of view.

Figure 2: Hyperpolarized gas MRI of the lungs which are un-corrected images from image set 11. Images present strong spatial variation in signal strength.
Figure 3: Hyperpolarized gas MRI of the lungs. Sensitivity corrected images from the data for image set 11, using 7 harmonics.
Figure 4: Hyperpolarized gas MRI of the lungs which are un-corrected images from image set 18. The red square indicates one of several ventilation defects.
Figure 5: Hyperpolarized gas MRI of the lungs. Sensitivity corrected images from the data for image set 18, using 7 harmonics. The red square indicates one of several ventilation defects.
Figure 6: A: ADC maps calculated with individual sensitivity maps for each image. B: ADC maps calculated with same sensitivity map for both images. Both from image set 18.
Figure 7: A: Image set 18 lung with a line through it that shows which row was selected.
B: Plotted profile of the given row.
Figure 8: A: Image set 11 lung with a line through it that shows which row was selected.
B: Plotted profile of the given row.

Figure 9: A typical Coil sensitivity map inversion matrix, from image set 18.
**Discussion**

As can be seen in the images above, the algorithm works quite well both when the original coil sensitivities are very inhomogeneous (figures 2 and figure 3) and when they are very homogeneous (figures 4 and figure 5). Both image sets show an improvement in image quality, with the images that are poor to begin with being the ones that show a larger improvement. This shows that the algorithm works well for images that have a wide range of coil sensitivity maps without any modification to the algorithm between the two datasets.

As can be seen in figures 2 and 3, there are two major changes between the corrected and the original images. The first is the more homogeneous lung parenchyma, which is the large kidney bean shaped portion of the lung which participates in gas exchange. This is the expected result, and this is the goal of the algorithm. The second change is the slight over gain of the major airways which is the contributing factor for making some of the corrected images look darker than they should. This is because the assumption that the high signal areas vary only because of the coil sensitivity at low frequency is violated by the high frequency and signal component of the major airways.

The most common qualitative metric used in hyperpolarized gas lung imaging is defect score. As discussed above, defect score is an important measure of lung function. Figures 2 through 5 show that the defects remain very visible, perhaps even more visible after the sensitivity correction than before. This is highlighted in figures 4 and 5 by red boxes surrounding one of many matching lung defects.

The major quantitative metric used is ADC mapping, as discussed above. Because of the importance of ADC maps, it is important that images with coil sensitivity correction produce similar, if not identical ADC maps. In practice, because the two images required for ADC map calculation are acquired at the same location, the coil sensitivity maps, and therefore inverse coil sensitivity maps should be the same. By averaging the results of the method on both images, coil sensitivity corrections were made which were identical in both images, and therefore the ADC map is within $3.5 \times 10^{-16}$ of the true map which is seen in figure 6b, which is 16 orders of magnitude below the quantization error in the original image. Therefore, this method shows that it does retain the desired quantitative information.

However, in order to assess the robustness of the method, we performed the ADC calculation on images which were corrected using their own sensitivity map with no
averaging. The absolute difference between the real and the corrected ADC maps are shown in figure 6a. The primary difference is in the trachea and the areas near the major airways. This is because the airways are the major region of difference between the two images used to construct the ADC image. This difference is typically quite small, with the typical maximum being about 0.2 per slice, with some central slices having a maximum close to 0.4. The typical ADC image ranges from about 0 to 2, so the change is large, but it is in a very limited region of the image, so the end effect is not that severe. The effect is not that surprising either, given the differences between the two images to begin with.

Another metric to determine the change from original to corrected was to plot line profiles of each image as seen in figures 7 and 8. The line profiles and images seen in figure 7 show that the correction equalizes the intensity from both lungs very well, and they clearly demonstrates that the high frequencies are unchanged. The images show the equalization to some extent, but the plots show that both lungs exhibit similar intensity. This is also shown to some extent in figure 8 (case 11) which shows the extremely heterogeneous lung. In this case the equalization does not fully correct the intensity difference, but it improves it by a significant amount. Looking at the images, this is clearly visible, with the lung on the left side of the image significantly darker than the other lung. In both cases, the equalization works very well, and the resulting images are improved, as the line profile shows.

The corrections shown in figures 3 and 5 are due to coil sensitivity corrections for each image; a typical example of which is shown in figure 9. As you can see the low gain areas are within the lung volume and higher gains are generally well outside the lung boundary. The triangular filter prevents the remaining noise from being over amplified in the area outside the lung volume. The reason the inverse matrix contains as much spatial complexity as it does is due to the non-uniform NMR emitting nature of the gas in the lung volume.

The algorithm is quite robust, as is seen in the results above. The major region of weakness is the incorrect assumption that all low frequency variations are due to coil sensitivity rather than actual signal variations. However, in most cases, the signal variation due to non-ideal coil sensitivities is far larger than the signal variation due to physiological and anatomical variations. Therefore the coil sensitivity correction does more good than harm in most, if not all, cases.
Conclusion

The images above show an improvement in the homogeneity of the signal intensity of the image, while preserving the quantitative measures and the means for qualitative scoring. It shows improvement despite the fact that it is based on a partially incorrect assumption. The assumption that the low spatial harmonics are the result of coil sensitivity and not subject signal is incorrect; because the lung is very dynamic and gravity dependent, the $^3$He is not distributed evenly throughout the lung. This uneven distribution is even caused and influenced by disease states similar to that of the lung defects, as discussed above.

The coil sensitivity corrected images clearly have a more homogeneous intensity than the non-corrected images. This holds true for the entire lung with the exception of the portions where the image has high intensity gradients. This is the case in the trachea and other airways and in the upper part of the images when the subject is in the supine position. These portions of the lung have an exceptionally high gain which is the major detractor from the sensitivity corrected images.

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The ADC maps, when using the same correction field, as would happen in reality, are practically identical to the correct ADC map. This was as expected, because the ADC calculation has inherent signal normalization, which is basically the equivalent of sensitivity normalization at high signal values. When each individual image was modified by its own correction map, the major difference was near the major difference points in the original images. While these differences are noticeable and significant from a quantization standpoint, the ADC maps are still quite similar.

The defects, as is easily seen in the figures 4 and 5, are not detrimentally effected by the coil sensitivity correction. In fact, the correction seems to improve the defect detection because previously dark sections of the lung become much brighter, and therefore it is far easier to find the defects in a qualitative manner. This is an excellent result because defects are an important part of hyperpolarized $^3$He MRI.

Because the image is distinctly improved by the coil sensitivity correction in most cases, the coil sensitivity correction is rarely detrimental to apply. However, the issue of extraordinarily bright portions of the image which contain higher frequency components is the major detriment of this method. The unwanted gains may be attenuated by using more harmonics, however this is not always desirable.

Overall, this is a good method to use when the image is unduly influenced by heterogeneous coil sensitivities, such as image set 18. It corrects for most of the
problems without requiring any additional scanning or time in the scanner. In its current state, it is not robust enough to automatically be applied to every lung image, but with some work to correct some of the issues with the execution, the theory could be easily applied as part of the general image reconstruction algorithm.
References


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