Refer to the figure above, a truck is to be backed-up to a loading dock located at coordinate (50, 0). (assume MKS unit) It is necessary that the truck is pointing straight outward from the loading dock. While backing up the truck, the driver is able to steer the front wheels within $\pm 30^\circ$ from the truck’s direction. Initially, the truck is parked within a parking lot whose lower left and upper right corners are (5, -95) and (95, -40) respectively, facing an arbitrary direction. At no time, the truck is allowed to move outside the area from (0, -100) (lower left corner) to (100,0) (upper right corner).

While backing, the truck moves backward at a constant speed $V$. The truck’s coordinate is measured at the center of its rear wheels. The distance between front and rear wheels is $L$. Two angles are defined: $\phi$ ($0 \leq \phi \leq 360^\circ$) is the angle between the truck’s direction and the positive y-axis of the coordinate, $\theta$ ($-30^\circ \leq \theta \leq 30^\circ$) is the angle between the front wheels’ direction and the truck’s direction.

The dynamics of this nonlinear system can be described by the following three equations:

\begin{align*}
    x(n+1) &= x(n) + dt \cdot V \cdot \cos \theta(n) \cos(\phi(n) - \pi/2) \\
    y(n+1) &= y(n) + dt \cdot V \cdot \cos \theta(n) \sin(\phi(n) - \pi/2) \\
    \phi(n+1) &= \phi(n) + dt \cdot V \cdot \sin \theta(n)/L
\end{align*}

Note that $x(n+1)$ is the value of $x$ measured $dt$ seconds later than $x(n)$. Also, note that it would be more convenient to represent $\phi(n)$ in radians (e.g. $\pi/2$) rather than in degrees (e.g. $90^\circ$).

A parking task is considered success if the following three conditions (a), (b), (c) are all met: (a) during the process, the truck stays within bounds. That is, $0 \leq x(i) \leq 100$, and $-100 \leq y(i) \leq 0$; (b) the final position of the truck, denoted by $x_f$ and $y_f$ (both with unit in meters) must satisfy: $|x - 50| \leq 1.5$, and $-1.5 \leq y \leq 0$ (c) the truck’s final direction is within $\pm 5^\circ$ from the desired orientation of $180^\circ$ (positive Y-axis direction). That is, $|\phi_f - 180| \leq 5^\circ$. 

1. (60 points)
Let $V = 3 \text{ m/s}$, $L = 1\text{ m}$, and $dt = 0.5 \text{ sec}$. If $(x(1), y(1)) = (40, -5)$, $\phi(1) = 45^\circ$, and $\{\theta(1), \theta(2), \cdots, \theta(5)\}$ are: $\{20^\circ, 30^\circ, -2^\circ, -10^\circ, -20^\circ\}$ respectively, find the truck’s positions $(x(n), y(n))$ and direction $\phi(n)$, for $n = 2$ to $n = 6$. List results in a table. Note that the result of $\phi(i)$ should be in degrees. But in computation, it is more convenient to be represented in radians.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x(n)$</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y(n)$</td>
<td>-5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi(n)$</td>
<td>45$^\circ$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Also, plot each of the truck’s positions, with the loading dock position $(50,0)$ indicated by a “*”.

(b) (5 points)
From eqs. (1), (2), and (3), it is clear that only two state variables are needed, namely, $x(n)$ and $\phi(n)$. The control input is the steering angle $\theta(n+1)$. For the range specified in the problem description above, use the Matlab routine fsgen.m to generate (i) 5 fuzzy sets for $x(n)$ over a discrete universe of discourse containing 25 evenly distributed points over the range of $x(n)$; (ii) 7 fuzzy sets for $\phi(n)$ with a discrete universe of discourse containing 91 points evenly distributed over $[0^\circ, 360^\circ]$, and (iii) 7 fuzzy sets for the output variable $\theta$ with a discrete universe of discourse containing 61 points evenly distributed over $[-30^\circ, 30^\circ]$. Do NOT print out the entire fuzzy sets data points. Rather submit three plots each containing all the fuzzy sets defined on a specific universe of discourse, with title and labels.

(c) (15 points)
Given a set of rules specified in a rule file called ruletrk.m, develop a fuzzy logic controller such that the truck is able to back up from any initial position within the parking lot to the loading dock such that $(x_{\text{final}}, y_{\text{final}}) = (50,0)$, and $\phi_{\text{final}} = 180^\circ$. Assume $V = 4\text{ m/s}$, $L = 1\text{ m}$, and $dt = 0.5\text{ sec}$. Submit the trajectory plot of a successful trial. An example is given below:

(d) (20 points)
Download the data file posdata.txt from the web site. Each row of this file contains the initial x-y coordinates of the truck and its corresponding angle in radian. Use the program you developed in part 1c to try out for each of the 100 initial position and angles.
Report those trials that failed. Also, draw a scatter plot of the initial coordinates or the truck of each of the 100 trials. Use "x" to indicate a failure and "o" to indicate a success.

(e) (15 points)
Tune the fuzzy logic controller, including fuzzy sets, and rule base to improve the performance of the truck backing system. The amount of improvement is measured by the reduction of the number of failed trials compared to that obtained in part 1d. Discuss what change you have made, and what outcome you obtained. Note that you are not to change the physical parameters such as the region defined as success backing, the truck's speed, steering angle range, etc. You can modify the definition of the fuzzy sets, as well as the set of fuzzy rules freely though.

2. (40 points) Use multi-layer perceptron neural network to perform pattern classification. The training data set $p_{\text{train}}$ and the feature vectors of the testing data set $p_{\text{test}}$ can be downloaded from the course home-page. The class labels of the testing set feature vectors are withheld. You score for this problem will be judged in part by your approach to solve the problem (20 points), and the remaining 20 points will be graded based on the relative classification rate on the testing results with respect to the best results submitted.

To document your approach, it should include:

(i) (10 points) An analysis of the data set and pre-processing procedures applied to the feature vectors.

(ii) (10 points) At least two different network structures you experimented, and your basis for selecting the network for testing. A table listing the training data sets, the target class label, and the class label obtained by the MLP classifier selected in part (ii).

(iii) (20 points) A table listing the testing feature vectors and your final classification results. The testing results must be printed on a separated page in the following format:

<table>
<thead>
<tr>
<th>feature 1</th>
<th>feature 2</th>
<th>feature 3</th>
<th>label</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>-0.43</td>
<td>1.7</td>
<td>0</td>
</tr>
</tbody>
</table>

Do NOT alter the order of the testing feature vectors.