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Channel / System Identification using Total Least Mean Squares (TLMS) Algorithm

Let \( Z(k) = [X(k) \mid d(k)] \) and \( \Delta Z(k) = [\Delta X(k) \mid \Delta d(k)] \) and \( W(k) = [w_1(k) \mid w_2(k) \mid \ldots \mid w_n(k)] \)

where \( Z(k) \) be the augmented vector of the input vector \( X(k) \) and the desired output \( d(k) \), and \( \Delta Z(k) \) be the augmented interference vector exists in input and desired signals.

Let \( \tilde{Z}(k) = Z(k) + \Delta Z(k) \) and \( \tilde{W}(k) = [W(k)^T \mid w_{n+1}(k)] \) and now we want to find the optimal solution by minimization of \( \min E \left\{ \| Z(k) \tilde{W}(k) \|^2 \right\} \) subject to \( \| W(k) \|^2 = \alpha \)

By the Lagrange expansion, we can reexpressed the problem to be

\[
J = \sum \left[ \tilde{W}^T(k) R \tilde{W}(k) \right] + \lambda \left[ \| \tilde{W}(k) \|_2 - \alpha \right], \quad \text{where} \quad R = E \left[ \tilde{Z}(k)^T \tilde{Z}(k) \right]
\]

and we can \( \tilde{W}(k) \) which minimize the cost function \( J \) by

\[
\frac{\partial J}{\partial \tilde{W}(k)} = 2R\tilde{W}(k) + \lambda = 0
\]

Hence, the solution of this optimization problem is the eigenvector, related to the smallest eigenvalue of the augmented correlation matrix.

Therefore, the recursive equation to find the optimum weight is

\[
\tilde{W}(k+1) = \tilde{W}(k) + \mu \left[ \tilde{W}(k) - \tilde{W}(k) \| R \tilde{W}(k) \|^2 \right], \quad \text{where} \quad R = E \left[ \tilde{Z}(k)^T \tilde{Z}(k) \right] = \frac{1}{M} \sum_{m=1}^{M} \tilde{Z}_m(k)^T \tilde{Z}_m(k)
\]

Since, in the TLMS algorithm, we compute the correlation matrix in each iteration. Let \( \tilde{Y}(k) = \tilde{Z}(k)^T \tilde{W}(k) \), then we can rewrite the equation as

\[
\tilde{W}(k+1) = \tilde{W}(k) + \mu \left[ \tilde{W}(k) - \| \tilde{W}(k) \|^2 \tilde{Y}(k) \tilde{Z}(k) \right]
\]