Lecture 24

Radial Basis Network (I)

Outline

- Interpolation Problem Formulation
- Radial Basis Network Type 1

Interpolation Problem Formulation

- Radial Basis function for interpolation:

Given $\{\mathbf{x}_i; 1 \le i \le K\}$ and $\{d_i; 1 \le i \le K\}$, find a function $F(\mathbf{x})$ that satisfies the interpolation condition:

$$F(\mathbf{x}_i) = d_i \qquad 1 \le i \le K \qquad (1)$$

One form of $F(\mathbf{x})$ is the radial basis function of the following form:

$$F(\mathbf{x}) = \sum_{i=1}^{K} w_i \, \phi(\|\mathbf{x} - \mathbf{x}_i\|) \qquad (2)$$

where $\{\mathbf{x}_i; 1 \le i \le K\}$ are the centers of the radial basis functions.

Solving Radial Basis Coefficients

- Substitute (1) into (2), we obtain a set of linear system of equations

$$M\,w = d \qquad (3)$$

where M = [M(i,j), $1 \le i, j, \le K$] is the *interpolation matrix*, $M(i,j) = \phi(\|\mathbf{x}_i - \mathbf{x}_j\|)$, $w = [w_1, w_2, \bullet\bullet\bullet, w_K]^t$, and $d = [d_1, d_2, \bullet\bullet\bullet, d_K]^t$. Given M and **d**, assuming the N centers are distinct, w can be solved as: $w = M^{-1}d$ if M is non-singular. If the $\phi(r) = (r^2 + c^2)^{-1/2}$, or $\phi(r) = \exp(-r^2/(2\sigma^2))$, it can further be shown that M is also positive definite.

An Example

- Let $\varphi(\|x-x_i\|) = (1-|x-x_i|)[u(x-x_i+1)-u(x-x_i-1)]$.
  Given $F(-1) = 0.2$, $F(-0.5) = 0.5$, and $F(1) = -0.5$.
  $F(x) = w_1\varphi(\|x-x_1\|)+w_2\varphi(\|x-x_2\|)+w_3\varphi(\|x-x_3\|)$
  $= w_1(1-|x+1|)\,[u(x+2)-u(x)] + w_2(1-|x+0.5|)\,[u(x+1.5) - u(x-0.5)] + w_3(1-|x-1|)\,[u(x) - u(x-2)]$. So,
- $F(-1) = w_1\bullet 1\bullet 1+w_2\bullet 0.5\bullet 1+w_3\bullet(-1)\bullet 0 = 0.2$
  $F(-0.5) = w_1\bullet 0.5\bullet 1+w_2\bullet 1\bullet 1+w_3\bullet(-0.5)\bullet 0 = 0.5$
  $F(1) = w_1\bullet(-1)\bullet 0+w_2\bullet(-0.5)\bullet 0+w_3\bullet 1\bullet 1 = -0.5$
- Solve for $w_1 = -1/15$, $w_2 = 8/15$, and $w_3 = -0.5$.

Example continued

- Suppose $\varphi(\|x-x_i\|) = \exp(-|x-x_i|^2)$, one has

$$\begin{bmatrix} 1 & e^{-|-1+0.5|^2} & e^{-|-1-1|^2} \\ e^{-|-0.5+1|^2} & 1 & e^{-|-0.5-1|^2} \\ e^{-|1+1|^2} & e^{-|1+0.5|^2} & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.5 \\ -0.5 \end{bmatrix}$$

  One can solve the $\underline{w}$ vector accordingly.

- Comparison with Parzen window: $h_3 = 1/\sqrt{3}$,

$$P(x) = \frac{1}{3}\sum_{i=1}^{3}\frac{1}{h_3}\varphi\left(\frac{x-x_i}{h_3}\right) = \frac{1}{\sqrt{3}}\left[e^{-3|x+1|^2} + e^{-3|x+0.5|^2} + e^{-3|x-1|^2}\right]$$

  – No weighting, and <u>no target values</u> of F(x) needed.

Example (Comparison)