Lecture 29
Introduction
to
Fuzzy Set Theory (I)

Outline

- What are Fuzzy Systems?
- Fuzzy logic applications
- Fuzzy set and fuzzy operations

Origin of fuzzy set theory

- Introduced by Lotfi Zadeh in 1965 as a way to manage complexity of systems.
- Using a term *principle of incompatibility*, Dr. Zadeh states

  "*As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics*."

[Zadeh, Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Trans. on Systems, Man, and Cybernetics,* Vol. 3, No. 1, 1973].

What are fuzzy sets?

- Fuzzy sets are functions that map each member in a set to a real number in [0, 1] to indicate the *degree of membership* of that member.
- The ambiguity of real world definitions

  *John is **OLD***

  *David is **TALL***

  How "OLD" is old? 40 years, 50, or 60?

  How "TALL" is tall? 5 feet, 6 feet, or 7 feet?
- **Every thing is a matter of degree**
- The "degrees" of being old or tall can be quantitatively illustrated using *quantified meaning.*

Fuzzy set examples

Excerpts from ***Wisconsin State Journal, 3/6/1994***

- A <u>weak</u> cold front spanning from Michigan through Texas and on to California will moderate the temperature <u>down to normal</u> in the <u>Midwest</u>. <u>Low</u> pressure over <u>northern</u> Texas will cause <u>scattered</u> rain and thundershowers from Missouri, <u>south</u> to Texas and <u>east</u> to Alabama.

Excerpts from ***New York Times*** (Zadeh)

- **<u>Dallas</u>**, Prices of crude oil, which have <u>edged higher</u> in <u>recent</u> weeks after being <u>remarkably stable</u> through <u>much of the year</u>, may <u>fluctuate</u> as much as a dollar a barrel in the <u>months ahead</u>, but <u>abrupt changes</u> are <u>not likely</u>, <u>many</u> analysts <u>believe</u>.

Why using Fuzzy Logic?

- A quantified framework to deal with the *IMPRECISE NATURE OF THE REAL WORLD* where conventional mathematical equations become intractable.
- A nature way to model expert's knowledge which are often imprecise in nature. For example,

  "In second-degree A-V block, the P-R interval may be either normal or prolonged, but some impulses are not propagated." [ECG processing text book]
- Model *inconsistent*, and *conflicting* opinions of multiple *experts* because the intersection of A and Not A is not a null set. Hence, conflict opinions can co-exist.
- Ability to deal with uncertainties, and unstructured knowledge.

Fuzzy Logic Applications

Replacement of a skilled human operator by a fuzzy rule based system

Sendal subway (Hitachi)

Cement kiln (F.L. Smith)

Elevator Control (Fujitec, Hitachi, Toshiba)

Sugeno's model car and model helicopter

Hirota's robot

Nuclear Reactor Control (Hitachi, Bernard)

Automobile automatic transmission (Nissan, Subaru)

Bulldozer Control (Terano)

Ethanol Production (Filev)

Appliance control – Washing machine, microwave ovens, rice cookers, vacuum cleaners, camcorders, TVs, thermal rugs, heaters.

Fuzzy Logic Applications

Replacement of a human expert by a fuzzy logic based decision making system

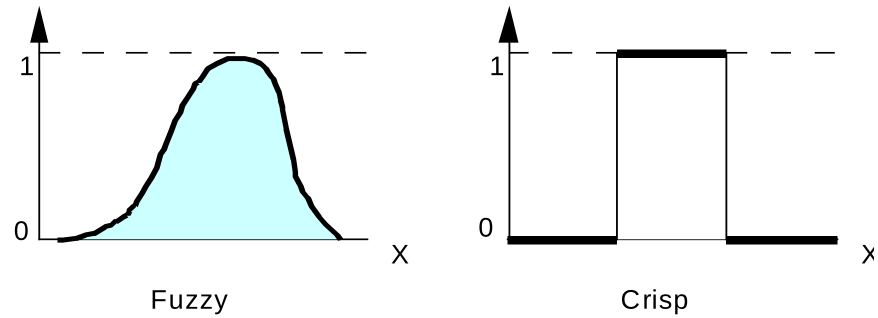
- Medical – CADIAG
- Securities (Yamaichi, Hitachi)
- Credit Worthiness (Zimmermann)
- Damage assessment (Yao, Hadipriono)
- Fault Diagnosis (Guangzhou)
- Production planning (Turksen)

Membership Function

- The membership function μ is a mapping from each element x in the universal set X to a real number.

  Crisp set – χ: X ∈ { 0, 1} (two elements)

  Fuzzy set – μ: X ∈ [0, 1] (an interval)



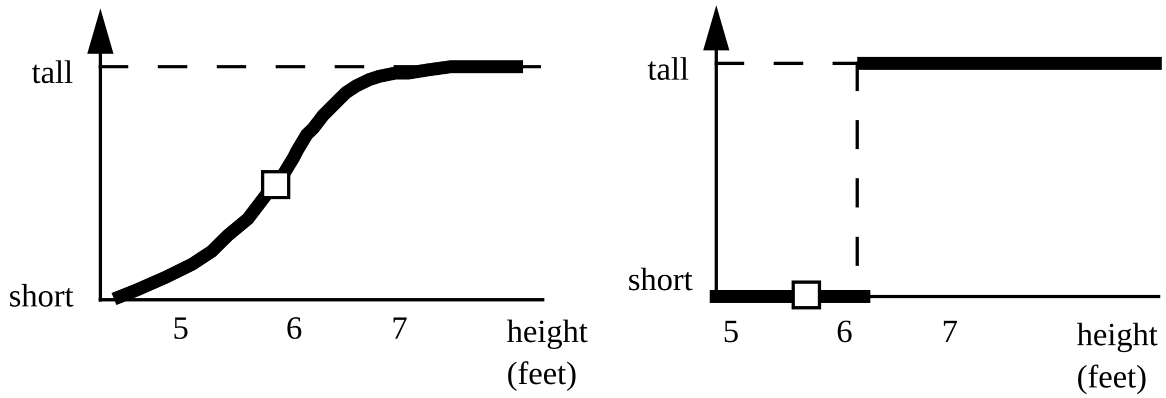
- Crisp set definition of the set of tall persons:

$$\chi_C(x) = \begin{cases} 1 & \text{if } x \in C; \\ 0 & \text{if } x \notin C. \end{cases}$$

where $C = \{x | x \le 6 \text{ feet}\}$.

Fuzzy Versus Crisp Sets

- Consider the notion of being a "TALL" person:



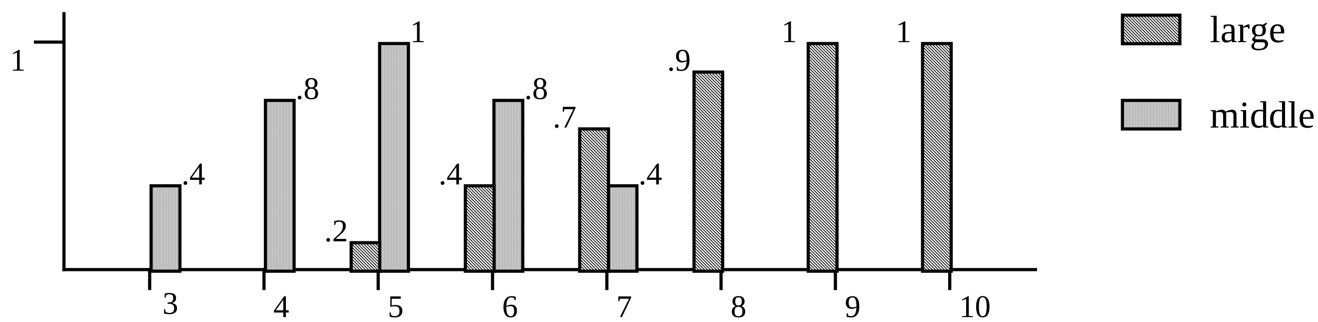
Fuzzy def. of tall   Crisp def. of tall

- In crisp set definition, $\chi_C(5'9'') = 0$, and $\chi_C(6'1'') = 1$
- In fuzzy set definition, $\mu_A(5'9'') = 0.6$, and $\mu_A(6'1'') = 0.8$

  In other words, although a person of 6'1" are more qualified to be called tall, someone with 5'9" will not be classified as being short.

Discrete Fuzzy Set SUPPORT

- If $X = \{x_1, x_2, \cdots, x_n\}$ be a set with discrete elements, the fuzzy set A defined on X can be expressed by
  $$A = \sum_{i=1}^{n} \mu_A(x_i)/x_i = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n$$
- Example – Let
  - large = 0.2/5 + 0.4/6 + 0.7/7 + 0.9/8 + 1/9 +1/10, and
  - middle = 0.4/3 + 0.8/4 + 1/5 + 0.8/6 + 0.4/7, then



Continuous Fuzzy Set Support

- On continuous support, fuzzy set can be expressed as:

  A = $\int_x \mu_A(x)/x$

- Use this expression, we have

  $A \cap B = \int_x [\mu_A(x) \wedge \mu_B(x)]/x$

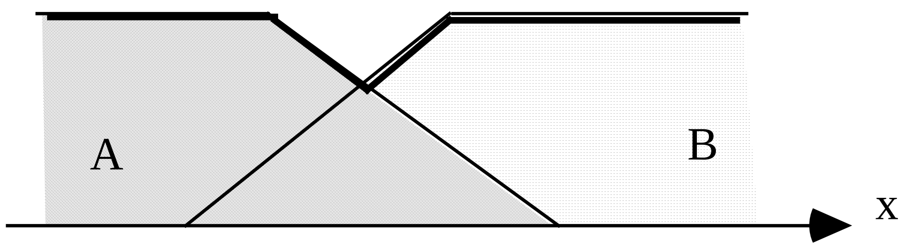
  $A \cup B = \int_x [\mu_A(x) \vee \mu_B(x)]/x$

- *Normal Fuzzy Set* $\int_x \mu_A(x) = 1.$

Fuzzy Set Operations

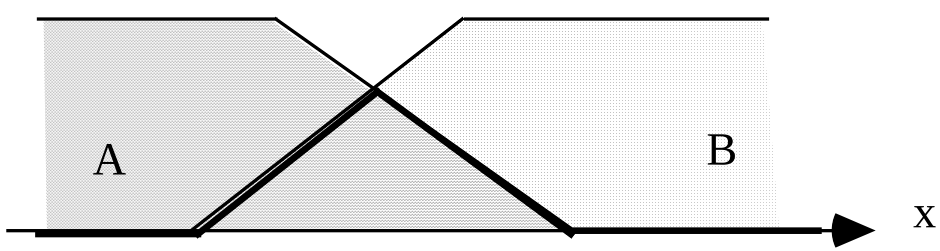
- Union of fuzzy sets D = A ∪ B:

  $\mu_D(x) = \max.\{ \mu_A(x),\ \mu_B(x) \} = \mu_A(x) \vee \mu_B(x)$



- Intersection of fuzzy sets D = A ∩ B

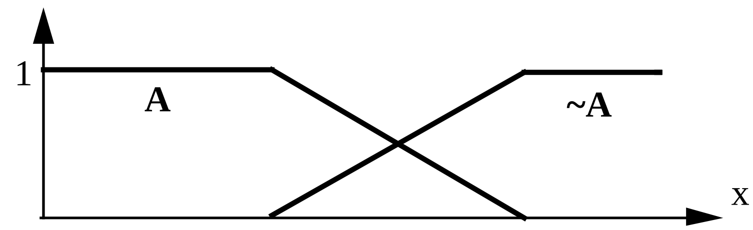
  $\mu_D(x) = \min.\{ \mu_A(x),\ \mu_B(x) \} = \mu_A(x) \wedge \mu_B(x)$



Fuzzy Set Operations

- Complementary: (~A)

  $\mu_{\sim A}(x) = 1 - \mu_A(x)$: degree of $x \in A$.



- Distinct Properties of Fuzzy Sets

  $A \cup (\sim A) \neq X$; $A \cap (\sim A) \neq \emptyset$.

  X can both be in A (with degree $\mu_A(x)$) and ~A (with degree $\mu_{\sim A}(x)$).

Alternative Fuzzy Operators

- *Compensatory operators* provide a weaker or less sensitive relationship among propositions when their truth values are widely separated.
- Some are commonly used compensatory operators

	Intersection	Union
Zadeh	Min(μ_A(x), μ_B(x))	Max.(μ_A(x), μ_B(x))
Product	μ_A(x)• μ_B(x)	μ_A(x)+ μ_B(x) – μ_A(x)• μ_B(x)
Bounded Sum	Max.(0, μ_A(x)+ μ_B(x)–1)	Min(1,μ_A(x)+ μ_B(x))