Lecture 30
Fuzzy Set Theory (II)

Outline

- Membership Function, Fuzzy Set Supports
- Fuzzy Set Operations, Alternative Operations
- Fuzzy Propositions
- Linguistic Variable Modifiers (Hedges)
- Fuzzy Relations

Fuzzy Proposition

- Fuzzy Proposition – Propositions that include fuzzy predicates. For example,
  - *It will be sunny today.*
  - *Dow Jones is closed higher yesterday.*
- Canonical Form – (Unconditional fuzzy proposition)

  x is A

  A (a fuzzy set) is a fuzzy predicate called the *fuzzy variable* or the *linguistic variable*. The values of a linguistic variable are words or sentences in a natural or synthetic language.

Fuzzy Proposition

- Note that a linguistic variable is a fuzzy (sub)set defined on a *Universe of discourse*. For example,

  Janet is young

  implies the *AGE* of Janet is Young. Here, Young is a fuzzy set defined on the axis "Age".
- Other fuzzy sets may be defined on the same universe include "Old", "Mid-age", etc.
- Age is a property of "Janet", and Young is a specific subset of "Age".

Linguistic Variable Modifiers

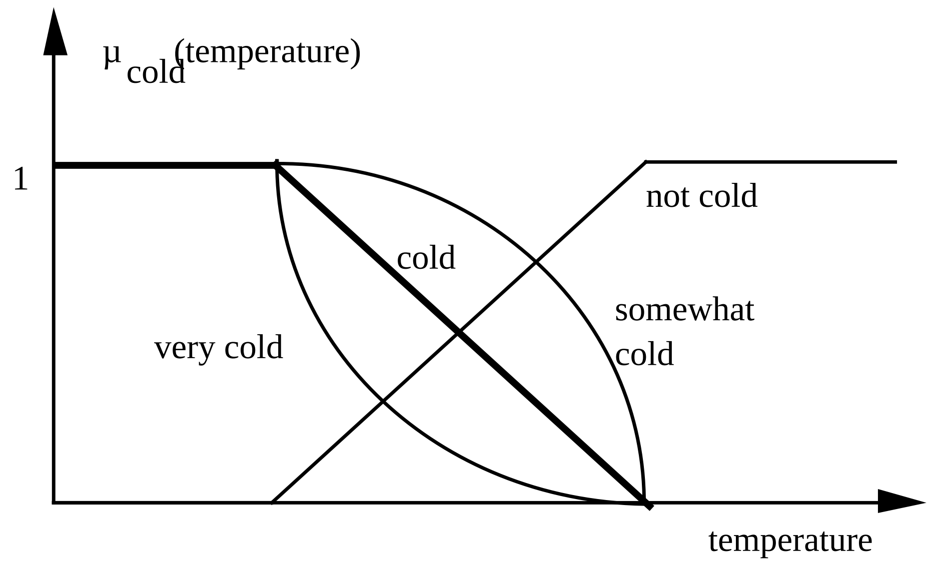
- *Modifiers* (hedges) are words like "extremely", "very" which changes the predicate. For example,

  "It is cold today" becomes "It is very cold today".

  Some possible implementations of modifiers are: Very, somewhat, Not, positively, etc.
- CONcentration and DILution –transform original membership function $\mu(x) \rightarrow \mu^n(x)$, $n > 1$ (concentration) and $n < 1$ (dilution).

Linguistic Variable Modifiers

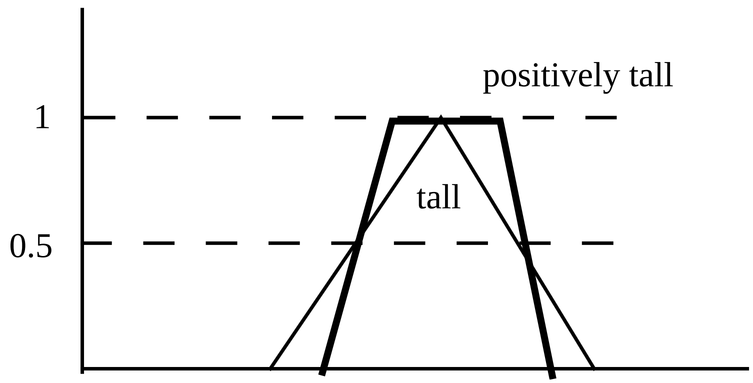
- Examples: VERY ($\mu^2(x)$), EXTREMELY ($\mu^3(x)$), SOMEWHAT, MORE_OR_LESS ($\mu^{0.5}(x)$)



Linguistic Variable Modifiers

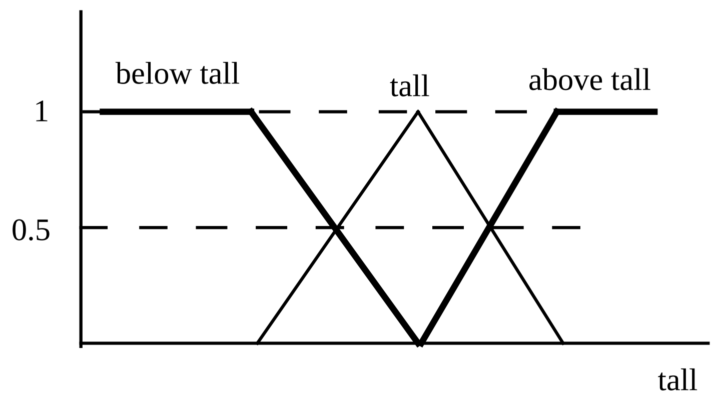
- INTensify – $\mu_{int}(x) = \begin{cases} n\mu^n(x);\ x \in A_\alpha \\ 1 - n\mu^n(x);\ x \notin A_\alpha \end{cases}$

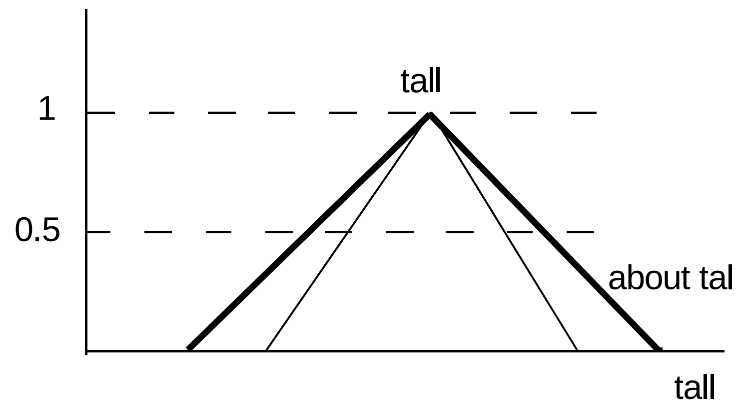
  $A_\alpha = \{x|\ \mu(x) \le \alpha\}$ is the α-cut of μ(x).
- For example, let n = 2, α = 0.5. The fuzzy sets Tall and POSITIVELY Tall are illustrated below:



Linguistic Variable Modifiers

- AROUND, ABOUT, APPROXIMATE – Broaden μ(x).
- BELOW, ABOVE – (see illustration below)



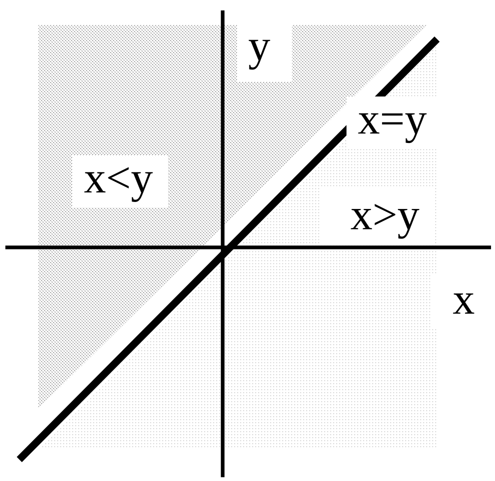


Fuzzy Relations

- Fuzzy relation R from set X to set Y is a fuzzy set in the direct product $X \times Y = \{(x,y) | x \in X, y \in Y\}$, and is characterized by a membership function

  $$\mu_R: X \times Y \rightarrow [0, 1]$$

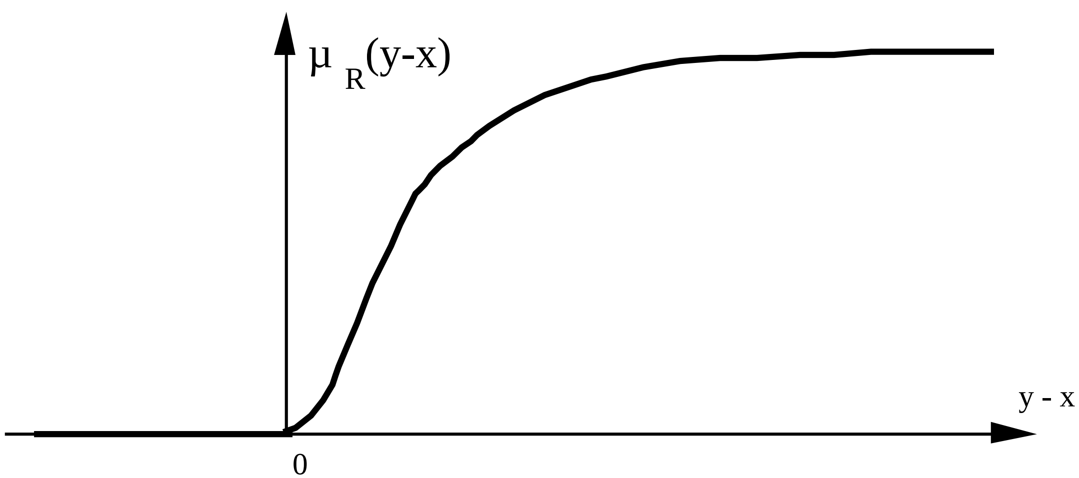
  When X = Y, R is a fuzzy relation on X.
- Relations defined in crisp logic:



Fuzzy Relation Example

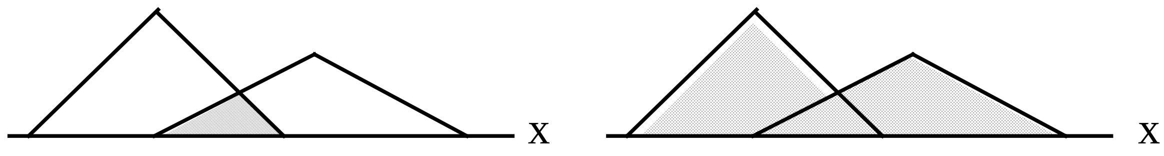
- Example. The relation y >> x can be defined as:

$$\mu_R(x,y) = \begin{cases} 0 & x \ge y; \\ \dfrac{1}{1+100(y-x)^2} & x < y. \end{cases}$$



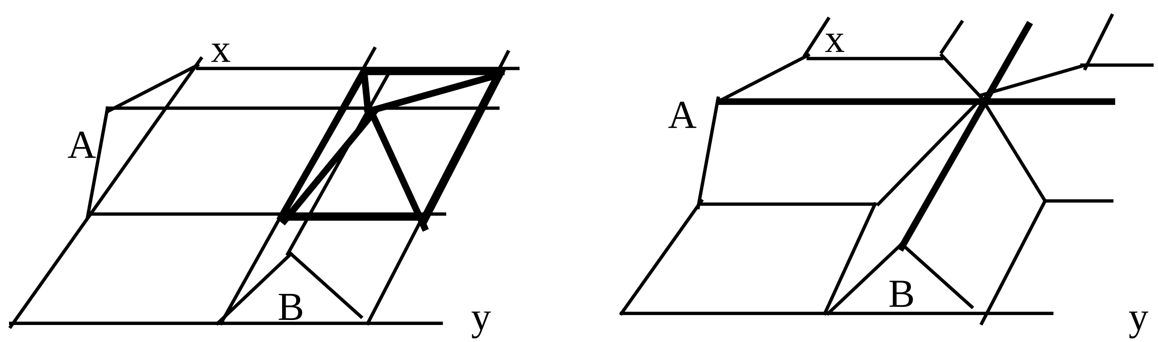
Other Fuzzy Relations

- "x is A" AND "x is B" = "x is (A AND B)”
  "x is A" OR "x is B" = "x is (A OR B)”



"x is A" AND "y is B" = "(x,y) is $A \times B$”

"x is A" OR "y is B" = "(x,y) is $A \times Y \cup X \times B$"



Fuzzy Matrices

- If both X and Y consists of finite, countable elements, then $\mu_R(x,y)$ can be represented by a matrix called a *Fuzzy matrix*.
- Example. X = {a, b, c}. Then a fuzzy relation R on X may be:

R = 0.2/(a,a) + 1/(a,b) + 0.4/(a,c) + 0.6/(b,b) + 0.3/(b,c) + 1/(c,b) + 0.8/(c,c) or

$$R = \begin{bmatrix} 0.2 & 1 & 0.4 \\ 0 & 0.6 & 0.3 \\ 0 & 1 & 0.8 \end{bmatrix}$$

Fuzzy Graph

- A *Fuzzy Graph* consists of nodes $\{x_i\} \approx \{y_j\}$, and arcs $\mu_R(x_i,y_j)$ from x_i to y_j.

  Example: Let $\{x_i\} = \{y_j\} = \{a, b, c\}$

$$R = \begin{bmatrix} 0.2 & 1 & 0.4 \\ 0 & 0.6 & 0.3 \\ 0 & 1 & 0.8 \end{bmatrix}$$