Lecture 31

Fuzzy Set Theory (3)

Outline

- Fuzzy Relation Composition and an Example
- Fuzzy Reasoning

Fuzzy Relation Composition

- Let R be a fuzzy relation in $X \times Y$, and S be a fuzzy relation in $Y \times Z$.
- The Max-Min composition of R and S, $R^o S$, is a fuzzy relation in $X \times Z$ such that

  $R^o S \leftrightarrow \mu_{RoS}(x,z) = \vee \{\mu_R(x,y) \wedge \mu_S(y,z) \}$

  $= \text{Max.} \{\text{Min.} \{\mu_R(x,y), \mu_S(y,z)\}\}/(x,z)$
- The Max-Product Composition of R and S, $R^o S$, is a fuzzy relation in $X \times Z$ such that

  $R^o S \leftrightarrow \mu_{RoS}(x,z) = \vee \{\mu_R(x,y) \bullet \mu_S(y,z) \}$

  $= \text{Max.} \{\mu_R(x,y) \bullet \mu_S(y,z)\}/(x,z)$

Fuzzy Composition Example

- Let the two relations R and S be, respectively:

R	y_1	y_2	y_3
x_1	0.4	0.6	0
x_2	0.9	1	0.1

S	z_1	z_2
y_1	0.5	0.8
y_2	0.1	1
y_1	0	0.6

- The goal is to compute R^{o}S using both Max-min and Max-product composition rules.

MAX-MIN Composition

$$R^{o}S = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0.9 & 1 & 0.1 \end{bmatrix} \circ \begin{bmatrix} 0.5 & 0.8 \\ 0.1 & 1 \\ 0 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 1 \end{bmatrix}$$

max{min(0.4,0.5), min(0.6, 0.1), min(0, 0)}

= max{ 0.4, 0.1, 0} = 0.4

max{min(0.4,0.8), min(0.6, 1), min(0, 0.6)}

= max{ 0.4, 0.6, 0} = 0.6

max{min(0.9,0.5), min(1, 0.1), min(0.1, 0)}

= max{ 0.5, 0.1, 0} = 0.5

max{min(0.9,0.8), min(1, 1), min(0.1, 0.6)}

= max{ 0.8, 1, 0.1} = 1

MAX-PRODUCT Composition

$$\text{RoS} = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0.9 & 1 & 0.1 \end{bmatrix} \circ \begin{bmatrix} 0.5 & 0.8 \\ 0.1 & 1 \\ 0 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.06 & 0.6 \\ 0.45 & 1 \end{bmatrix}$$

$$\max\{0.4 \cdot 0.5,\ 0.6 \cdot 0.1,\ 0 \cdot 0\} = \max\{0.02, 0.06, 0\} = 0.06$$

$$\max\{0.4 \cdot 0.8,\ 0.6 \cdot 1,\ 0 \cdot 0.6\} = \max\{0.32,\ 0.6,\ 0\} = 0.6$$

$$\max\{0.9 \cdot 0.5,\ 1 \cdot 0.1,\ 0.1 \cdot 0\} = \max\{0.45,\ 0.1,\ 0\} = 0.45$$

$$\max\{0.9 \cdot 0.8,\ 1 \cdot 1,\ 0.1 \cdot 0.6\} = \max\{0.72,\ 1,\ 0.06\} = 1$$

Fuzzy Reasoning

- Comparing crisp logic inference and fuzzy logic inference

Crisp logic	Mary is 22 years old Dana is 3 years older than Mary . ——————————————— Dana is (22 + 3) years old

Translation –

Age(Mary) = 22

(Age(Dana),Age(Mary)) = Age(Dana)–Age(Mary) = 3

$\therefore$ Age(Dana) = Age(Mary) + 3 = 22 + 3 = 25

Fuzzy Reasoning

Fuzzy logic	Mary is Young Dana is much older than Mary . Dana is (Young o Much_older)

Translation –

Age(Mary) = Young (Young is a fuzzy set)

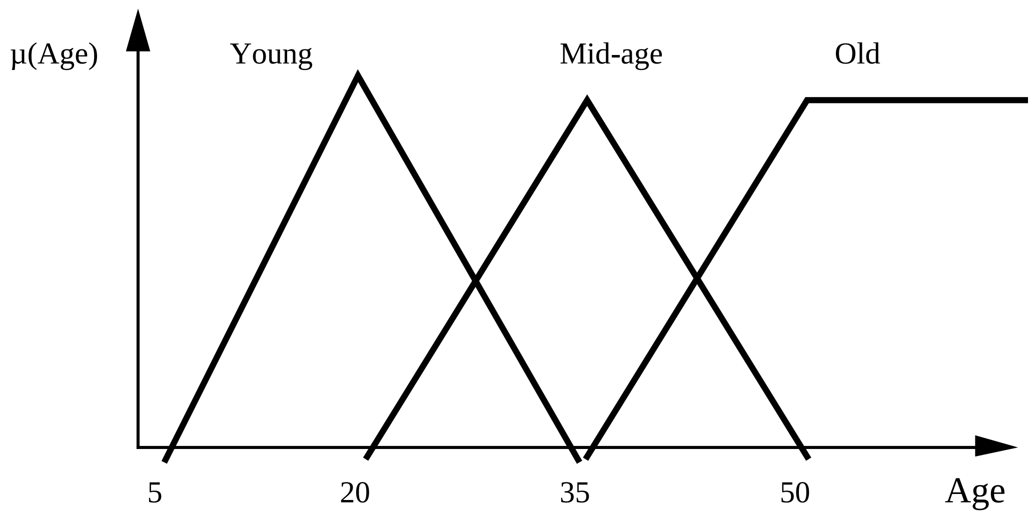
(Age(Dana),Age(Mary)) = Much_older (a relation)

$\therefore$ Age(Dana) = Young $\circ$ Much_older

– a composite relation!

Fuzzy Reasoning (cont'd)

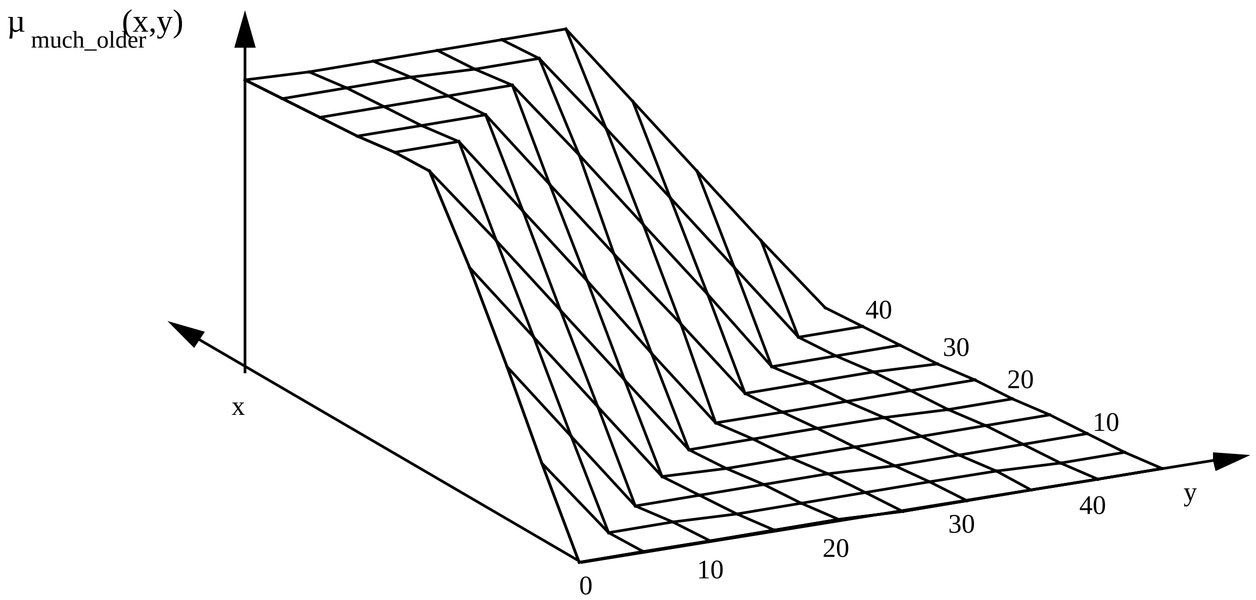
- $\mu_{Age(Dana)}(x) = \vee \{\mu_{young}(y) \wedge \mu_{much\_older}(x,y) \}$
  The universe of discourse (support) is "Age" which may be quantified into several overlapping fuzzy (sub)sets: Young, Mid-age, Old with the following definitions:



Fuzzy Reasoning (cont'd)

- Much_older is a relation which is defined as:

$$\mu_{much\_older}(x,y) = \begin{cases} 1 & x - y > 20, \\ \frac{1}{20}(x-y) & 0 < x - y \le 20, \\ 0 & x \le y. \end{cases}$$



Reasoning Example

For each fixed x, find

$$\mu_{Age(Dana)}(x) = \max(\min(\mu_{young}(y), \mu_{much\_older}(x,y))):$$