Lecture 32

Fuzzy Set Theory (4)

Outline

- Fuzzy Inference
- Fuzzy Inference method
  - Zadeh's formula
  - Composition rule
  - Mamdani's formula
  - Multiple fuzzy rules

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Fuzzy Inference

- An IF-THEN-ELSE rule (conditional proposition) is *activated* if part or all of its preconditions (the IF part) are satisfied to a degree such that it produces a non-zero grade value of its conclusions (the THEN part).

Example. Consider two rules in the rule base

- Rule 1. IF angle is small, THEN force is large
- Rule 2. IF angle is medium, THEN force is medium.
- Suppose μ_{small}(Angle) = 0.4, and μ_{medium}(Angle) = 0.7. Then it is likely both rule 1 and rule 2 will be activated.

QUESTION: How to find the fuzzy set which describe the fuzzy variable "force" as a results of firing rules 1 & 2?

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Implication Formula (Zadeh)

- In conventional mathematical logic (Crisp Logic), the logic predicate "A implies B" is evaluated as:

  $A \rightarrow B$ = If A then B = $\sim A \vee B$ = If ~B then ~A

  Example. All human are mortal

  = If "x is human" then "x is mortal"

  = "x is NOT human" OR "x is mortal"

  = If "x is NOT mortal" then "x is NOT human"

- In Fuzzy Logic, the implication is a relation which can be defined (due to Zadeh) similar to crisp logic case as:

$$R_a = A \rightarrow B = \int_{u \times v} \{1 \wedge (1 - \mu_A(u) + \mu_B(u))\} \,/(u, v)$$

Note that $\sim A \times 1 - \mu_A(u)$. The term "$1 \wedge$" is to limit the membership function from greater than unity.

Composition of Inferences

- Consider the following composition of inferences:

  Premise 1 If x is A then y is B

  Premise 2 x is A’ .

  Conclusion y is B'

  GOAL: Given A', Find B’

  Answer: Use Max-min composition rule, we have

$$B' = A' \circ (A \rightarrow B), \quad \text{or}$$

$$\mu_{B'}(v) = \{ \mu_{A'}(u) \wedge \mu_{A\rightarrow B}(u,v)\}$$

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An Example

Consider the propositions below:

If it is in the north of Wisconsin, then it is cold

John lives in the FAR north of Wisconsin .

Conclusion: It is VERY cold

- Define fuzzy sets:

north = 0.1/Madison + 0.5/Dells + 0.7/Greenbay + 1/Superior;

cold = 1/20 + 0.9/35 + 0.4/50 + 0.2/65

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Example cont'd

- If north (A) then cold (B) = north $\rightarrow$ cold =

north\cold	1	0.9	0.4	0.2
0.1	1	1	1	1
0.5	1	1	0.9	0.7
0.7	1	1	0.7	0.5
1	1	0.9	0.4	0.2

This relation is found by the equation:

$$\mu_{north \varnothing cold}(u,v) = 1 \wedge \{1 - north(u) + cold(v)\}$$

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Example cont'd

- Far_north (A') = north^2 = 0.01/Madison + 0.25/Dells + 0.49/Greenbay + 1/Superior
- Use Max-min composition rule,

$$B' = [0.01\ \ 0.25\ \ 0.49\ \ 1] \circ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & .9 & .7 \\ 1 & 1 & .7 & .5 \\ 1 & .9 & .4 & .2 \end{bmatrix} = [\,1\ \ 0.9\ \ 0.49\ \ 0.49]$$

- However, a Modus *Ponens* Property is not satisfied:

  if A' = A, then

$$B' = [0.1\ \ 0.5\ \ 0.7\ \ 1] \circ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & .9 & .7 \\ 1 & 1 & .7 & .5 \\ 1 & .9 & .4 & .2 \end{bmatrix} = [\,1\ \ 0.9\ \ 0.7\ \ 0.5] \neq B!$$

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Mamdani's Formula

- Zadeh's implication formula does not satisfy the modus poenes property.
- Mamdani's Method

  $R_c = A \rightarrow B = \int_{U \times V} [\mu_A(u) \wedge \mu_B(v)]$ /(u, v)

  $\mu_{A\rightarrow B}(u) = \mu_A(u) \wedge \mu_B(v)$ "$\wedge$" can be Min. or Product
- Composition Rule of Inference: $B' = A' \circ (A \rightarrow B)$,

  $\mu_{B'}(v) = \{ \mu_{A'}(u) \wedge \mu_{A\rightarrow B}(u,v)\} = \{ \mu_{A'}(u) \wedge [\mu_A(u) \wedge \mu_B(v)]\}$

  $= \{ [\mu_{A'}(u) \wedge \mu_A(u)] \wedge \mu_B(v)\} = \{ [\mu_{A'}(u) \wedge \mu_A(u)]\} \wedge \mu_B(v)$
- If A is normalized such that max(A) = 1, then clearly, when A' = A (i.e. $\mu_{A'}(u) \wedge \mu_A(u) = \mu_A(u)$), B' = B.

Mamdani's Method (Example)

- Example. (same example as the previous one)

north = 0.1/1 + 0.5/2 + 0.7/3 + 1/4;

cold = 1/20 + 0.9/35 + 0.4/50 + 0.2/65

Far_north (A') = north^2 = 0.01/1 + 0.25/2 + 0.49/3 + 1/4

max{min(A, A')} = max{ min{(0.1,0.01), (0.5,0.25), (0.7,0.49), (1,1)}} = max{0.01, 0.25, 0.49, 1} = 1.

Hence B' = min(B, 1) = B even A' $\neq$ A!

Observation:

We may not get the *Very cold* conclusion using Momdani's method. (Or maybe northern Wisconsin isn't so cold anyway!)

Mamdani's Method (Example)

Example. (Continuous fuzzy variable)

If u is A,

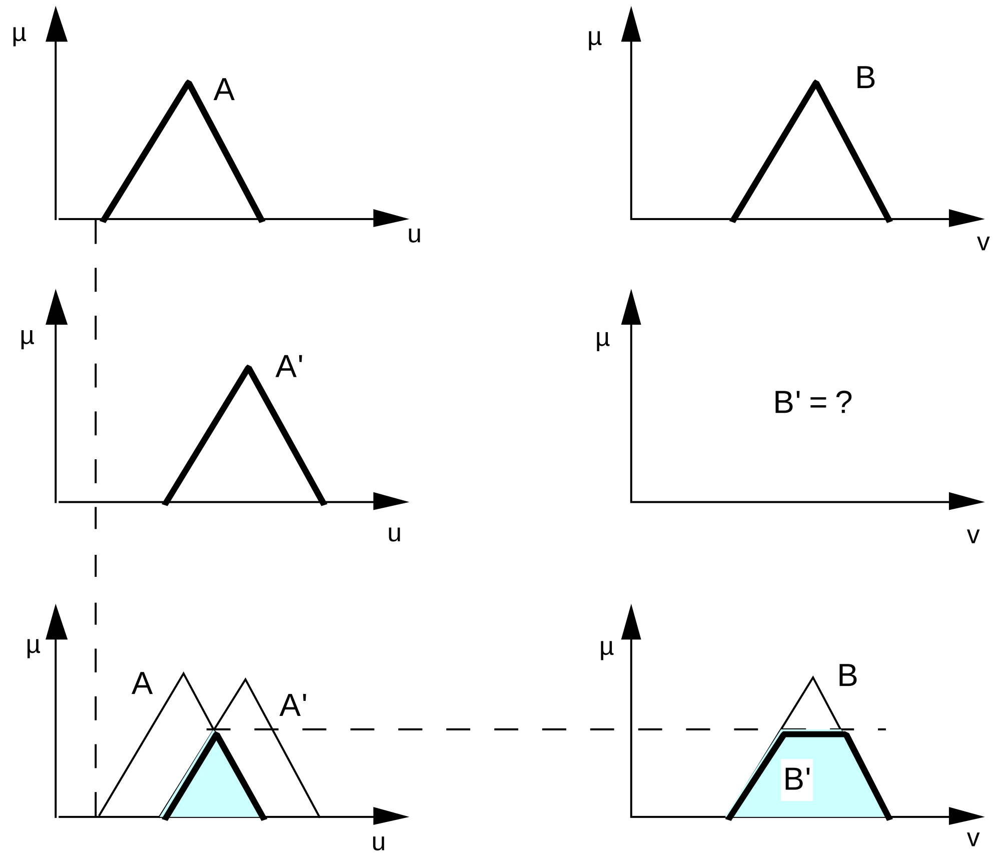
then v is B

If u = A'

then v = ?

B' = Min.{B,

Max. (A,A')}



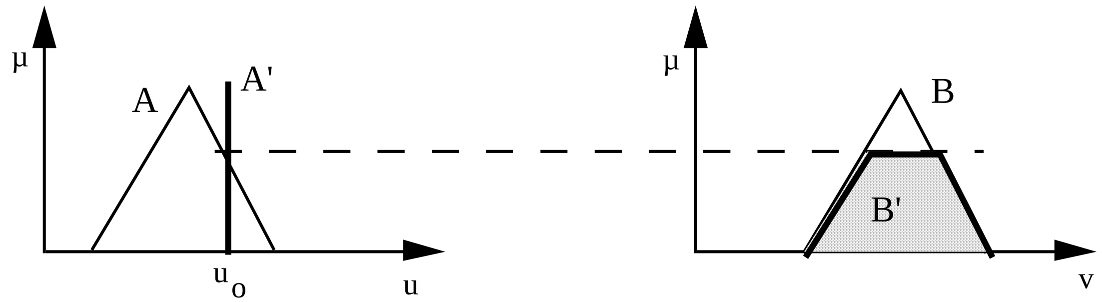
Example cont'd

- Crisp Input value: $u = u_o$

  When $A' = 1/(u = u_o)$, we have

  $\text{Max}(\text{Min}(A, A')) = \mu_A(u_o)$, and

  $\mu_{B'}(v) = \max(\mu_B(v), \mu_A(u_o))$



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Multiple Fuzzy Rules

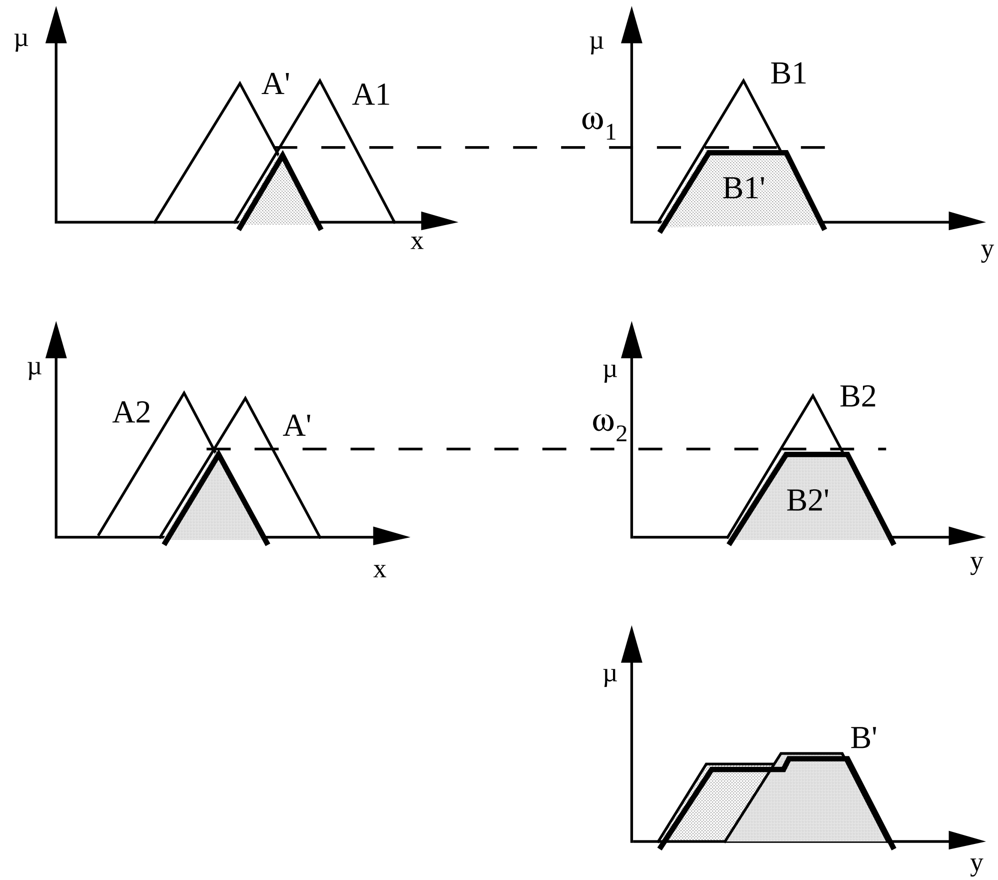
Rule #1.

If X is A_1, then Y is B_1.

Rule #2.

If X is A_2, then Y is B_2.

Query: If X is A', then Y = B' = ?



ω_1, ω_2 are called compatibility for each of the antecedent conditions of the rules and the input.

Fuzzy Inference Example

Query #2

When X is x_o, what is Y = B' = ?•

Max-product composition rule is used instead of the *max-min* composition rule used in the first query.