Lec 34
Fuzzy Logic Control (II)

Outline

- Control Rule Base
- Fuzzy Inference
- Defuzzification
- FLC Design Procedures

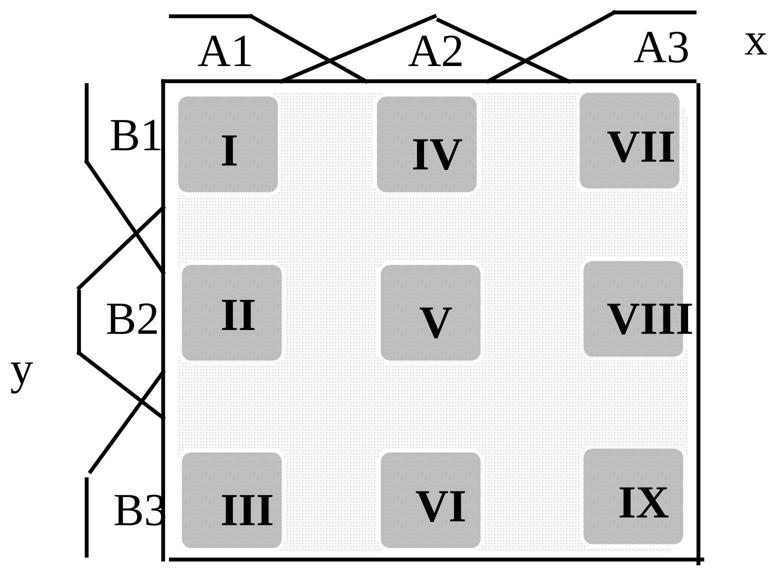
Control Rule Base

- General form of rule:

  IF x_1 is A_1 AND • • • AND x_M is A_M, Antecedent

  THEN y_1 is B_1 AND • • • AND y_N is B_N, Consequent

- The rule base form a *fuzzy partition* of the multi-dimensional space.



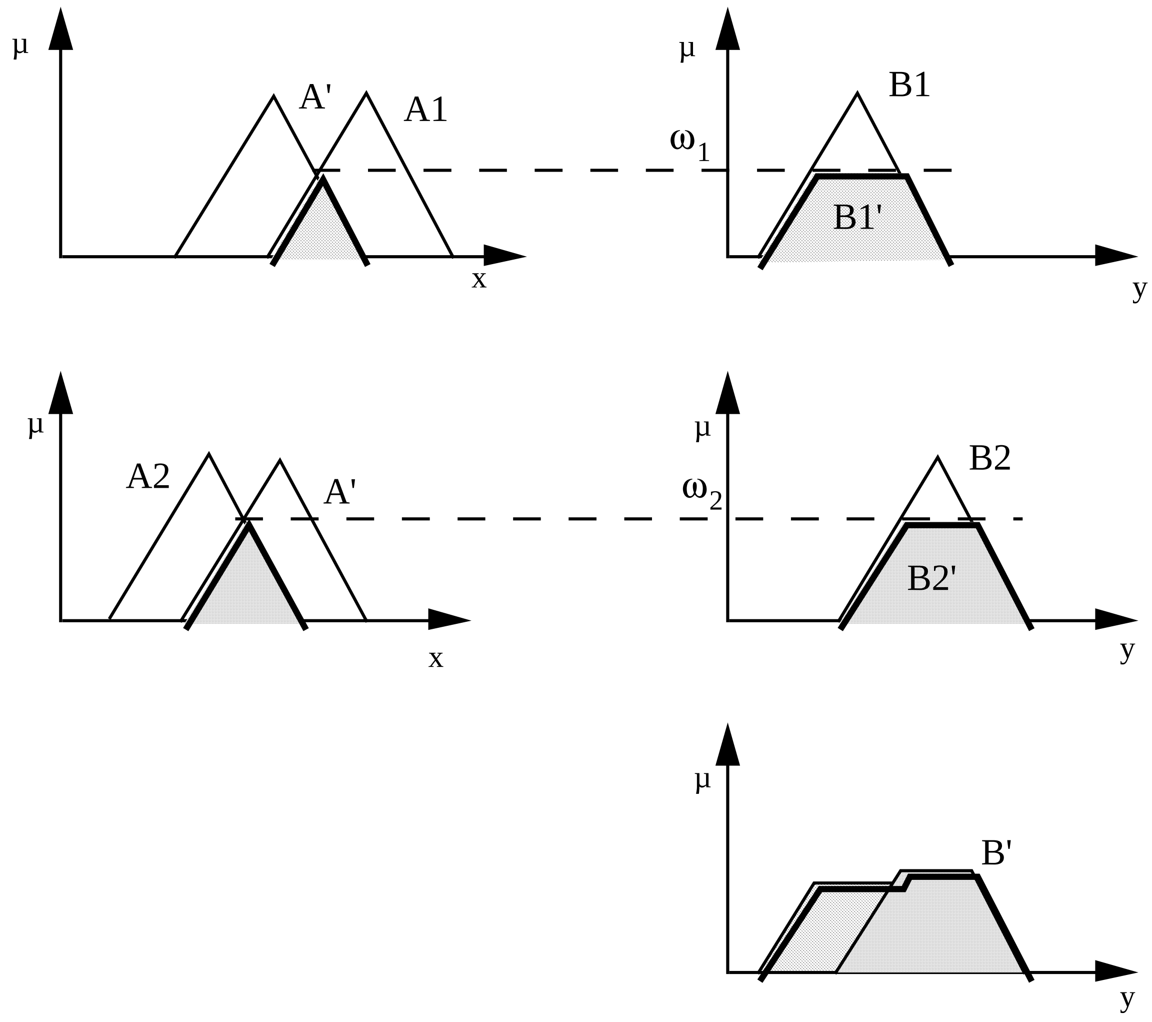
For example: (Rule IV) IF x is A2, AND y is B1, THEN u is C4.

Total 9 rules can be defined. Often not all possible rules need to be defined explicitly.

Inference Engine

- When input variables are fuzzified, *Each* rule in the rule base will try to determine its degree of activation using min-max or correlation-max method.
- For rules which has a non-zero activation value, the output fuzzy variables will be combined (fuzzy union) yielding a resultant fuzzy set.

Fuzzy Inference



Defuzzification

(1) The *Center of Area* (COA) Method

$$y^*_{COA} = \frac{\int y \cdot \mu_{B'}(y)dy}{\int \mu_{B'}(y)dy}$$

(2) The *Mean of Maximum* (MOM) Method

Define $B'_{max} = \{y \mid \mu_{B'}(y) \geq \mu_{B'}(y') \text{ for } y' \in B'\}$ to be the set of y in B' where $\mu_{B'}(y)$ reaches maximum.

$$y^*_{MOM} = \frac{\int_{B'_{\max}} y dy}{\int_{B'_{\max}} dy}$$

Defuzzification

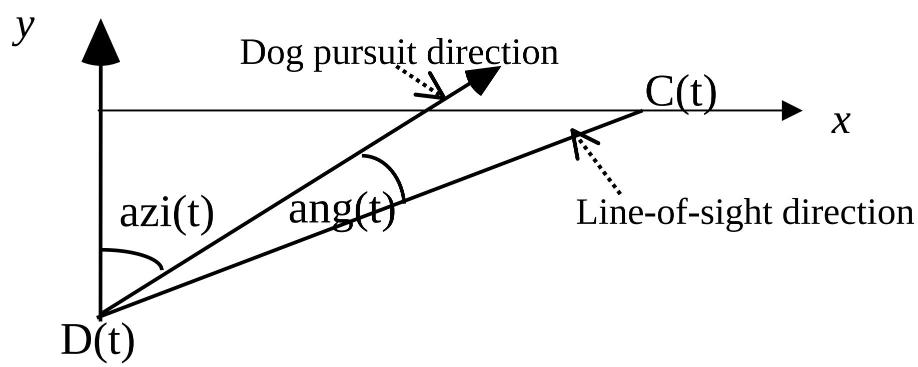
(3) When outputs are functions of the inputs

For example, $Y = f_i(X)$ where $1 \le i \le n$ is the index for the rules. Then

$$y^* = \frac{\sum_{i=1}^{n} \omega_i f_i(x_o)}{\sum_{i=1}^{n} \omega_i}$$

Rule Weighting ω_i – The weighting of each rule may be set so that some rules carry more influence than others. The weighting will take effect during defuzzification.

A Dog Chases Cat Example



- Cat travels east bound at a constant speed v.
- Dog travels at a pursuit direction that has an angle of azi(t) from north (clockwise direction is positive angle) with a constant speed w (> v).
- The angle between the line-of-sight direction from dog to cat and the dog's current pursuit direction is ang(t).
- Goal: to line up the dog's pursuit direction to that of the line-of-sight direction. Since the dog runs faster (in this example), it will eventually catch up the cat!

Dog Chases Cat Control Model

- Dependent dynamic variables (functions of state variables):

  $ang(t) = atan\,(x_{cat}(t)-x_{dog}(t))/(y_{cat}(t)-y_{dog}(t)) – azi(t)$
- Control goal:
  - Given ang(t), to minimize ang(t+1) by applying appropriate control input dz(t) to update azi(t+1).
- Variable types:
  - State variables C(t), D(t) (cat’s and dog’s positions) and azi(t) shall remain crisp variables.
  - dz(t) (control input) and ang(t) (derived from state variables) will be fuzzified to facilitate fuzzy logic control.

Control Law Strategy

- Control Law: (proportional control)

  $$dz(t+1) = K \cdot ang(t)$$

- Question: How to choose the most appropriate value of the proportion constant K?

  Observe that this system is nonlinear in nature, conventional PID design technique will be difficult to apply.

  Extensive simulation, trial-and-error will be needed!

FLC Design Procedures

Step 1. Establish a mathematical model.

- Identify State variables (input to the controller) and control input variables (output of the controller) for the model.
- Characterize the type of these variables (fuzzy or crisp?) . Specify the control objective.

- Plant:
  - 5 state variables: $C(t) = (x_{cat}(t), y_{cat}(t))$, $D(t) = (x_{dog}(t), y_{dog}(t))$, azi(t), one (control) input variable dz(t)

$x_{cat}(t+1) = x_{cat}(t) + v\bullet 1; \qquad y_{cat}(t+1) = y_{cat}(t)$

$x_{dog}(t+1) = x_{dog}(t) + w\bullet\sin(azi(t+1));$

$y_{dog}(t+1) = y_{dog}(t) + w\bullet\cos(azi(t+1));$

$azi(t+1) = azi(t) + dz(t)\bullet 1$

FLC Design Procedure

Step 2. Fuzzification:

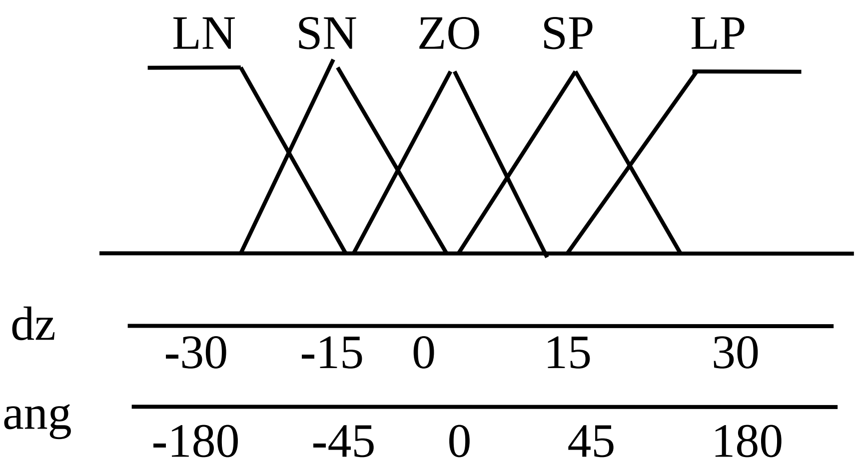
Fuzzy decomposition of the range of each fuzzy variable.

- Specify the range of each fuzzy variable, and then devise a set of linguistic variables.
- It is important that the supports of adjacent linguistic variables overlap so that more than one fuzzy rules may be fired.
- The shape of membership function may be chosen empirically based on pdf of experimental data or heuristically.

Fuzzification for DCC example

- Both ang(t) and dz(t) will share the same set of linguistic variables: LN, SN, ZO, SP, LP
- The dynamic ranges of the supports (universe of discourse) of these two fuzzy variables however, are different.
  - dz(t): -30^o to 30^o
  - ang(t): -180^o to 180^o
- These choices may be due to physical constraints and other prior knowledge. They may need fine-tuning too.

- In this example, the range difference is determined by the proportional control law constant K.



FLC Design Procedures

Step 3. Devise fuzzy control rules

Rules can be represented conveniently as a table if there are two input fuzzy variables.

A simple rule base for the DCC example that has only one input fuzzy variable ang(t) and one control (output) variable dz(t+1).

The scalar in front of each rule indicates the relative weighting of that rule.

(1.0) If ang(t) is LN, Then dz(t+1) is LN.

(1.0) If ang(t) is SN, Then dz(t+1) is SN.

(1.0) If ang(t) is ZO, Then dz(t+1) is ZO.

(1.0) If ang(t) is SP, Then dz(t+1) is SP.

(1.0) If ang(t) is LP, Then dz(t+1) is LP.