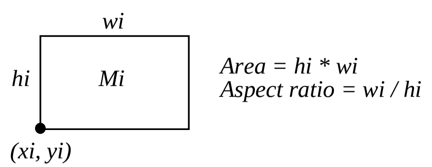
Floorplanning by Mathematical Programming

- Sutanthavibul, Shragowitz, and Rosen, "An analytical approach to floorplan design and optimization," 27th DAC, 1990.
- Notation:
  - w_i, h_i: width and height of module M_i.
  - (x_i, y_i): coordinate of the lower left corner of module M_i.
  - $a_i \le w_i/h_i \le b_i$: aspect ratio w_i/h_i of module M_i. (Note: We defined aspect ratio as h_i/w_i before.)
- Goal: Find a mixed **integer linear programming (ILP)** formulation for the floorplan design.
  - **Linear** constraints? Objective function?



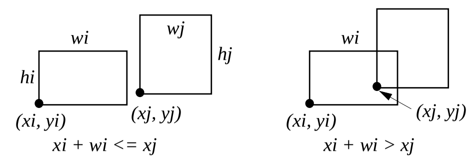
Nonoverlap Constraints

- Two modules M_i and M_j are nonoverlap, if at least one of the following linear constraints is satisfied (cases encoded by p_{ij} and q_{ij}):

		p_{ij}	q_{ij}
M_i to the left of M_j:	$x_i + w_i \le x_j$	0	0
M_i below M_j:	$y_i + h_i \le y_j$	0	1
M_i to the right of M_j:	$x_i - w_j \ge x_j$	1	0
M_i above M_j:	$y_i - h_j \ge y_j$	1	1

- Let W, H be upper bounds on the floorplan width and height, respectively.
- Introduce two 0, 1 variables p_{ij} and q_{ij} to denote that one of the above inequalities is enforced; e.g., $p_{ij} = 0, q_{ij} = 1 \Rightarrow y_i + h_i \le y_j$ is satisfied.

$$\begin{aligned} x_i + w_i &\le x_j + W(p_{ij} + q_{ij}) \\ y_i + h_i &\le y_j + H(1 + p_{ij} - q_{ij}) \\ x_i - w_j &\ge x_j - W(1 - p_{ij} + q_{ij}) \\ y_i - h_j &\ge y_j - H(2 - p_{ij} - q_{ij}) \end{aligned}$$



Cost Function & Constraints

- Minimize $Area = xy$, **nonlinear!** (x, y: width and height of the resulting floorplan)
- How to fix?
  - Fix the width W and minimize the height y!
- Four types of constraints:
  1. no two modules overlap ($\forall i, j : 1 \le i < j \le n$);
  2. each module is enclosed within a rectangle of width W and height H ($x_i + w_i \le W, y_i + h_i \le H, 1 \le i \le n$);
  3. $x_i \ge 0$, $y_i \ge 0, 1 \le i \le n$;
  4. $p_{ij}, q_{ij} \in \{0, 1\}$.
- w_i, h_i are known.

Mixed ILP for Floorplanning

Mixed ILP for the floorplanning problem with rigid, fixed modules.

$$\min \quad y$$

subject to

$$\begin{aligned} x_i + w_i &\le W, & 1 \le i \le n \quad &(1) \\ y_i + h_i &\le y, & 1 \le i \le n \quad &(2) \\ x_i + w_i &\le x_j + W(p_{ij} + q_{ij}), & 1 \le i < j \le n \quad &(3) \\ y_i + h_i &\le y_j + H(1 + p_{ij} - q_{ij}), & 1 \le i < j \le n \quad &(4) \\ x_i - w_j &\ge x_j - W(1 - p_{ij} + q_{ij}), & 1 \le i < j \le n \quad &(5) \\ y_i - h_j &\ge y_j - H(2 - p_{ij} - q_{ij}), & 1 \le i < j \le n \quad &(6) \\ x_i, y_i &\ge 0, & 1 \le i \le n \quad &(7) \\ p_{ij}, q_{ij} &\in \{0, 1\}, & 1 \le i < j \le n \quad &(8) \end{aligned}$$

- Size of the mixed ILP: for n modules,
  - # continuous variables: $O(n)$; # integer variables: $O(n^2)$; # linear constraints: $O(n^2)$.
  - Unacceptably huge program for a large n! (How to cope with it?)
- Popular LP software: LINDO, lp_solve, etc.

Mixed ILP for Floorplanning (cont'd)

Mixed ILP for the floorplanning problem: rigid, freely oriented modules.

$$\min \quad y$$

subject to

$$x_i + r_i h_i + (1-r_i) w_i \le W, \quad 1 \le i \le n \quad (9)$$
$$y_i + r_i w_i + (1-r_i) h_i \le y, \quad 1 \le i \le n \quad (10)$$
$$x_i + r_i h_i + (1-r_i) w_i \le x_j + M(p_{ij} + q_{ij}), \quad 1 \le i < j \le n \quad (11)$$
$$y_i + r_i w_i - (1-r_i) h_i \le y_j + M(1 + p_{ij} - q_{ij}), \quad 1 \le i < j \le n \quad (12)$$
$$x_i - r_j h_j + (1-r_j) w_j \ge x_j - M(1 - p_{ij} + q_{ij}), \quad 1 \le i < j \le n \quad (13)$$
$$y_i - r_j w_j - (1-r_j) h_j \ge y_j - M(2 - p_{ij} - q_{ij}), \quad 1 \le i < j \le n \quad (14)$$
$$x_i, y_i \ge 0, \quad 1 \le i \le n \quad (15)$$
$$p_{ij}, q_{ij} \in \{0,1\}, \quad 1 \le i < j \le n \quad (16)$$

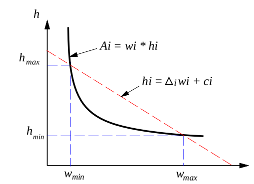
- For each module i with free orientation, associate a 0-1 variable r_i:
  - $r_i = 0$: 0° rotation for module i.
  - $r_i = 1$: 90° rotation for module i.
- $M = \max\{W, H\}$.

Flexible Modules

- Assumptions: w_i, h_i are unknown; area lower bound: A_i.
- Module size constraints: $w_i h_i \ge A_i$; $a_i \le \frac{w_i}{h_i} \le b_i$.
- Hence, $w_{min} = \sqrt{A_i a_i}$, $w_{max} = \sqrt{A_i b_i}$, $h_{min} = \sqrt{\frac{A_i}{b_i}}$, $h_{max} = \sqrt{\frac{A_i}{a_i}}$.
- $w_i h_i \ge A_i$ nonlinear! How to fix?
  - Can apply a first-order approximation of the equation: a line passing through (w_{min}, h_{max}) and (w_{max}, h_{min}).

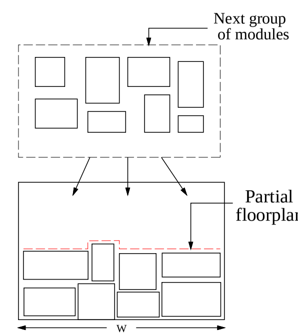
$$h_i = \Delta_i w_i + c_i \qquad /*\ y = mx + c\ */$$
$$\Delta_i = \frac{h_{max} - h_{min}}{w_{min} - w_{max}} \qquad /*\ slope\ */$$
$$c_i = h_{max} - \Delta_i w_{min} \qquad /*\ c = y_0 - mx_0\ */$$

  - Substitute $\Delta_i w_i + c_i$ for h_i to form linear constraints (x_i, y_i, w_i are unknown; Δ_i, Δ_j, c_i, c_j can be computed as above).



Reducing the Size of the Mixed ILP

- Time complexity of a mixed ILP: exponential!
- Recall the large size of the mixed ILP: # variables, # constraints: $O(n^2)$.
  - How to fix it?
- Key: Solve a partial problem at each step (successive augmentation)
- Questions:
  - How to select next subgroup of modules? ⇒ linear ordering based on connectivity.
  - How to minimize the # of required variables?



Reducing the Size of the Mixed ILP (cont'd)

- Size of each successive mixed ILP depends on (1) # of modules in the next group; (2) "size" of the partially constructed floorplan.
- Keys to deal with (2)
  - Minimize the problem size of the partial floorplan.
  - Replace the already placed modules by a set of covering rectangles.
  - # rectangles is usually much smaller than # placed modules.