1. **(10 points) System Diagnosis**

Consider a system consisting of 4 units as shown in Figure 1. Clearly this system is not two-fault one-step diagnosable. Prove this by listing two fault sets which give rise to the same syndromes. Show the fault sets in two figures and write their syndromes or list them in the form of a table.

![Figure 1: System architecture (system with 4 units)](image-url)
2. **(15 points) System Diagnosis**

Consider a system with 5 units shown in Figure 2. You will notice that the edges marked in this graph are not directed. It is because this figure represents a test graph of a system that uses “comparison testing”. In this model two units compare their results and the outcome of the result (0 or 1) is shown on the edge. The outcomes are defined as follows. If both units are fault free then the outcome is 0. If exactly of the two units, which compare their results, is faulty then the outcome is 1. If both units are faulty then the outcome can be either 0 or 1 (i.e. X). Note in this case the syndrome will be a 5 bit vector and if none of the units in the system is faulty then the syndrome will be 0 0 0 0 0.

Using any method you like (such as logic reasoning, dictionary of possible syndromes, etc.) prove or disprove that the 5 unit system shown in Figure 2 is one-step 2-fault diagnosable.

![Figure 2: System architecture (system with 5 units for comparison testing)](image)

3. **(15 points) System Diagnosis**

From the bound discussed in class it is evident that a single loop system with 9 units is not 4 fault sequentially diagnosable. Prove this by constructing fault sets, containing no more than 4 faults, in a single loop system with 9 units, and show that they can give rise to same syndromes, thus making diagnosis impossible.
4. (10 points) Error correcting coding
   Problem 2 on page 102 of the text.
   (Note: My notation has been different from the book, hence the (72,8) Hamming code mentioned in the book is same as (72,64) Hamming code in the notation I used)

5. (10 points) Hamming code and Parity coding
   Problem 4 on page 102 of the text.

6. (15 points) ECC and Solving Markov model
   Problem 12 on page 103 of the text. Note: You will have to read about RAID-1 in the text.

7. (10 points) Byte error detection
   H matrix of a (12,8) code is shown below. Prove that it is a SED-SBD code for b = 4. (note it can not correct single error, why?)

   \[
   \begin{array}{cccc|cccc|cccc}
   1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
   0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\
   1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
   1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
   \end{array}
   \]

8. (15 points) Cyclic code Note: You may have to read about polynomial multiplication and division in the text.
   (a) Draw an LFSR that can multiply an input polynomial \( D(x) \) by a polynomial \( g(x) = 1 + x + x^3 \). Note this will be a 3 bit register with exclusive-ORs at appropriate places.
   (b) Multiply the following two polynomials with \( g(x) \) and show your work. Note all additions are modulo 2.
      1) \( D(x) = x + x^3 \)
      2) \( D(x) = 1 + x + x^3 \)
   (c) Simulate the LFSR for the above inputs. You must show all the intermediate states of the shift register. Note that simulation will require seven steps in each case.