Ch. 1 - Viscosity & the Mechanics of Momentum Transport

- **Hookean Momentum Transport**
  - Newton's Law of Viscosity: \[ 
  \tau_x = -\mu \frac{dy}{dz} 
  \]
  - Shear stress is a force/unit area.
  - but, fluid flowing in the x-direction is acting on a surface w/ a y = const. face.
  - force is exerted by fluid of lesser y on fluid of greater y.

- There are 2 molecular stresses: 1 is from pressure, the other is from viscous stress.
  \[ 
  \tau_{ij} = \rho \delta_{ij} + T_{ij} 
  \]
  \( \delta_{ij} = 1 \) if \( i = j \) but \( \delta_{ij} = 0 \) if \( i \neq j \).
  - \( \rho \delta_{ij} \) is the sum of the molecular stresses bearings the stress from pressure.
  - \( T_{ij} \) is the sum of the molecular stresses bearing the stress from viscous stresses.

- **Convective Momentum Transport**
  - Momentum is transported by the bulk motion of the fluid.
  - Convective momentum transport = \( \frac{\partial u}{\partial x} \) (momentum carried in x-direction).

- **Combined Momentum Flux**
  - Sum of the molecular + convective terms.
  \[ 
  \dot{\rho} = \nabla \cdot \mathbf{T} \]
  - So, \( \dot{x} = \rho \frac{\partial u}{\partial x} + \nabla \cdot \mathbf{T} \)
  - \( \dot{y} = \frac{\partial u}{\partial y} + \nabla \cdot \mathbf{T} \)

- **Momentum equation**, meaning it's divided by area (area which is why it all balances it's multiplied by an area term).

Ch. 2 - Shell Momentum Balance:

\[ 
\Phi_i \cdot \text{Area} = \Phi_{\text{ext}} \cdot \text{Area} + \text{Force of gravity} = 0 
\]

\( (5g \text{ W}) \)
Choosing Shell:
In all the momentum X-for problems done in this class, vel. is assumed to depend on only 1 coordinate. So, if velocity is a \( f(x) \) \( x \) (4-not y or z), then the shell has taken on so that it has a thickness of \( dx \). It's dimension in the y or z direction or finite (not zero).

Ex: Falling Film Problem (p. 42).

![Diagram showing a falling film with labels for momentum and velocity.]

\[ \phi_{in}\cdot Wx \bigg|_{x=0} - \phi_{ex}\cdot Wx \bigg|_{x=L} + \phi_{ex}\cdot LW \bigg|_{x=L} - \phi_{in}\cdot LW \bigg|_{x=0} + \rho gLWdx = 0 \]

\[ \frac{\phi_{in} - \phi_{ex}}{L} = \frac{d\phi_{in}}{dx} + \rho g = 0 \]

(Don't forget the negative sign)

- Component of \( y \)-direction:
  \[ q_y = g \cos \theta \]

- Component of \( z \)-direction:
  \[ q_z = g \sin \theta \]

- \( \phi_{in} \), \( \phi_{ex} \) \( x \) = 0, \( x \) = \( L \), \( g \) = \( g \cos \theta \)

\[ \frac{d^2 u}{dx^2} = - g \cos \theta \]

\[ \frac{d^2 v}{dx^2} = - g \sin \theta \cdot x + c \]

\[ v(0) = g \cos \theta \cdot \frac{1}{2} x+c+\text{c_1} \]

BC: \( \text{At} x=0, v=x=0 \)

- \( \psi(x) = g \cos \theta \cdot \frac{1}{2} x+c_1 \) (since \( \psi \) is constant, shear stress is zero).
From BC1: \( \gamma_1 = 0 \), so \( \frac{d\gamma}{dx} = 0 \)

\[
\frac{d\gamma}{dx} = -\frac{\delta g \cos \theta}{M} \cdot 0 + C_1 \implies C_1 = 0
\]

From BC1:

\[
\gamma_2 = 0 = -\frac{\delta g \cos \theta}{M} \cdot \frac{x^2}{2} + C_2 \implies C_2 = \frac{\delta g \cos \theta}{M} \cdot \frac{\delta^2}{2}
\]

\[
\gamma_3 = \frac{\delta g \cos \theta - \delta^2}{2M} \left[ 1 - \left( \frac{x}{\delta} \right)^2 \right]
\]

- To calculate \( \text{vol} <V_3> \), integrate \( \gamma_3 \) over the cross-sectional area, then divide by the area:

\[
<V_3> = \frac{\int_{0}^{\delta} \gamma_3 \, dx \, dy}{\text{area}}
\]

- Force exerted by fluid on the solid surface:

\( \text{integrate} \gamma_3 \) over the surface area.

However, since the fluid (regarded as incompressible) is exerting a force on the solid (regarded as rigid), the sign should be positive.

\[
F_y = \int_{x=\delta}^{\infty} \left[ \gamma_3 \right] \, dy \, dx
\]

- \( \text{Ex: Flow through circular tube & Anulus:} \)

- See Section 2.3 & 2.4

- Note: for a vertical tube, flow is due to a combination of pressure drop and gravity, which is why the \( \delta \) term is used.

Flow has to go in the direction to high \( P \) to low \( P \), which is why in this system:

\[
P = \rho \cdot g \cdot z
\]

\( \text{We need } P_g \text{ to be } > P_o \text{ for flow to go downward.} \)
In the system: $P = p + \frac{g}{\rho}$ so that $\rho \leq \rho^*$.

Note: a common mistake is to write the shell balance for circular tube "virtual problem"

\[ \phi \left[ 2\pi rL - \phi \right] - \phi \left[ 2\pi rL + \ldots \right] = 0 \]

This is wrong. We think should really be $= \text{r}_r + \ldots$

\[ \phi \left[ 2\pi rL \right]_{r} - \phi \left[ 2\pi rL \right]_{\text{r}_r + \ldots} = 0 \]

**Ch3 Eqs of change for Isoth. Systems:**

- Continuity equation derives from a mass balance:
  \[ \frac{\partial \rho}{\partial t} = -(\rho \cdot \nabla) \rightarrow \text{flux} \]  
  \[ \rho = \text{const.} \]  
  \[ 0 = \nabla \cdot \rho \cdot \nabla \]

- Eqn of motion derives from a momentum balance:
  \[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \cdot \mathbf{v}) = [\nabla \cdot (\rho \cdot \mathbf{v})] - \rho \cdot \{ \nabla \cdot \mathbf{v} \} + \frac{g}{\rho} \]

- Simplification to EOM:
  - $g \cdot \rho \cdot \text{are const.}$
    \[ \frac{\partial \rho}{\partial t} \mathbf{v} = -\nabla \cdot (\rho \cdot \nabla \mathbf{v}) + g \mathbf{v} \]

  Navier–Stokes Eqn
  (see $$\frac{\partial \rho}{\partial t}$$ left)

  - Substantial time derivative:
    \[ \frac{\partial \mathbf{v}}{\partial t} = \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot \mathbf{v} \]

  - Vel is $v$, small $\rightarrow$ can neglect LHS
    \[ 0 = -\nabla \cdot (\rho \cdot \nabla \mathbf{v}) + g \mathbf{v} \]

  Stokes flow eqn.
- Ch 6 - Interphase Transport in Soil Systems

Def of Friction Factor:

\[ F = A K f \]

- Flow in a conduit:

\[ A = \text{wetted surface}, \ \text{For circular pipe, } A = \pi R L \]
\[ K = \frac{1}{2} \rho V^2 \]
\[ \therefore \ F_e = (2\pi RL) \left( \frac{1}{2} \rho V^2 \right) f \]
\[ \therefore F_e = (A - 2R) \cdot \pi R^2 \]

\[ \text{set } F_e = \text{to each other } \Rightarrow f = \frac{1}{4} \left( \frac{1}{2} \left( \frac{2R - 2R}{2\pi RL} \right) \right) \]

- Flow around a submerged object:

\[ A = \text{X-sectional area}, \ \text{For sphere, } A = \pi R^2 \]
\[ K = \frac{1}{2} \rho V^2 \quad \text{[approach]} \]
\[ \therefore \ F_e = (\pi R^2) (\frac{1}{2} \rho V^2) f \]
\[ \therefore F_e = \text{force of gravity on sphere - buoyant force on sphere} \]
\[ F_e = \frac{4}{3} \pi R^3 \text{ \text{g m} \cdot \text{g} - \frac{1}{6} \pi R^2 \text{ \text{g m} \cdot \text{g}}} \]

\[ \text{set } F_e = \text{to each other } \Rightarrow \]
\[ f = \frac{4}{3} \frac{g D}{V^2} \left( \frac{\text{Sphr} - \text{Fluid}}{\text{Fluid}} \right) \]
- Find value of friction factor for a given of Re → find value of Re to know whether flow is laminar or turbulent.
- Use eqns & figures in section 6.2 to get value of f.
- Mean hydraulic radius $R_h$ → usually used for turbulent flow in pipes rather than laminar flow.

$$R_h = \frac{\frac{A}{2}}{2 \pi r}$$

For ex: for a circular tube:

$$R_h = \frac{\pi D^2}{4 \pi} = \frac{D}{2}$$

**Ch.7 - Macroscopic Balances for Isothermal Flow:**

- The most frequently used eqn in Ch.7 is probably 7.5-10

$$\sum \left( \frac{1}{2} \left( v_x + v_y \right) \right)^2 + \sum g(z_s - z_x) = \int_{v_1}^{v_2} \frac{1}{2} dp = \Delta \rho = \sum \left( \frac{1}{2} v^2 \right) R_h + f \left( \frac{1}{2} v^2 \right)$$

(evaluated at plane 1)
(evaluated at plane 2)

→ choose plane 1→2 to make the problem as easy as possible.

**Energy Transfer**

**Chs. 9, 10, 11, 14, 15, 16**
Mass Transfer

Ch. 11 - Diffusion & Henry's Law of Mass Transfer:

- Molecular Mass Transfer:
  \[ \dot{m}_i = S_i (Y_A - Y_i) = -S_i \frac{\partial \rho}{\partial \rho} \frac{\partial \rho}{\partial x_i} \rightarrow (\dot{m}_g = -S_g \frac{\partial \rho}{\partial \rho} \frac{\partial \rho}{\partial y_g}) \]
  \[ J_A = C_A (Y_A - Y_i) = -C_A \frac{\partial \rho}{\partial \rho} \frac{\partial \rho}{\partial x_A} \rightarrow (\dot{J}_g = -C_g \frac{\partial \rho}{\partial \rho} \frac{\partial \rho}{\partial y_g}) \]

- Molar Mass:
  \[ \gamma = \frac{\sum n_i \omega_i Y_i}{\sum n_i \omega_i} \rightarrow \text{for binary system: } \gamma = \omega_A Y_A + \omega_B Y_B \]
  \[ \gamma = \sum n_i \omega_i x_i \rightarrow \gamma = x_A Y_A + x_B Y_B \]

- Mole Fraction:
  \[ n_A = \frac{\dot{m}_A}{\gamma} \]
  \[ x_A = \frac{n_A}{\dot{m}} \]

\[ \text{total mols + molar fractions must sum to 1, so for a binary system:} \]
\[ \text{so mass fraction of } \frac{\text{A}}{\text{B}} \rightarrow \text{mass fraction of } \frac{\text{A}}{\text{B}} \]
\[ x_A + x_B = 1 \]

- Convective Transfer:
  mass is transferred by bulk motion of fluid:
  \[ \text{convective mass flux rate} = f \dot{m}_B \]
  \[ \text{mole flux} = C_B \dot{m}_B \]

- Combined Mass Transfer:
  \[ \text{Sum of molecular + convective terms} \]
  \[ \dot{m} = \dot{m}_A + \dot{m}_B \]
  \[ N_{AB} = \dot{m}_A + C_B \dot{m}_B \]

- Ch. 12 - Concentration:
  \[ N_{AB} = -C_A B \frac{\partial \rho}{\partial y_A} + \gamma (N_{AB} + N_{BA}) \text{ for Binary System} \]
  \[ \text{(molecular molar flux + convective molar flux)} \]
In many problems, A is diffusing in B, which is stagnant, so:

\[ N_A = -\frac{C_A^0}{A} \frac{dX_A}{dt} \]

- **Shell balance:**

\[
\text{rate in} \quad A \text{ in} \quad \text{rate out} \quad \text{rate gained} = \frac{d}{dt} (\text{mass A})
\]

(some eqn can be written for A)

- **Chemical rxn's:**

  - **Homogeneous rxn occurs throughout a vol**
    
    \[ k_A = k_{\text{vol}} \frac{[A]^{[0]}}{[A]^n} \]
    
    rate occurring throughout a vol

  - **1st ord rxn: \( k \left[ \frac{mol}{L \cdot s} \right] \)**

  - **Heterogeneous rxn's occurring on surface of a catalyst**

    \[ N_A = k_{\text{area}} \frac{[A]^{[0]}}{[A]^n} \]
    
    rate occurring on a surface

    \[ 1^\text{st ord} \text{ rxn: } k \left[ \frac{mol}{cm \cdot s} \right] \]

- **Ch. 19 - Eqn's of Change:**

  - See Section 19.1 for eqns.