Curvature-Dependent Conductance Resonances in Quantum Cavities

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Summary. Conductance in planar quantum cavities is known to exhibit resonances which depend on the dimensions of the cavity and the applied electric and magnetic fields. We demonstrate that these resonant features are also highly sensitive to the curvature of the cavity, and suggest that these curved quantum cavities, fabricated on flexible substrates, have potential applications as MEMS components or highly-responsive sensors.

Quantum-mechanical properties of non-planar two-dimensional (NP2D) systems have been discussed for a number of years, and multiple approaches have been developed [1,2]. This topic has received renewed interest, however, since the development and refinement of fabrication methods for NP2D devices in the quantum regime [3,4]. There has been much recent theoretical work describing their novel transport properties [5], and experimental results are only beginning to come in [6].

In this paper, we investigate ballistic transport in a flexible 2D electron system, and show that the conductance is highly sensitive to the structure's local curvature. In planar quantum cavities, it is well established [7] that low-field ballistic conductance will exhibit resonant features. Resonant maxima and antiresonant minima in conductance are attributed to quantum-mechanical states with very high and very low transmission (respectively) between contacts. In flexible quantum cavities, we show that the existence and location of such resonances can be tuned through the manipulation of the cavity's curvature.

We consider a simple quantum cavity that can be rolled into a partial cylinder, and study the effect of curvature on its conductance (Fig. 1). The dimensions of this tethered cylindrical cavity are 300x200 nm, with 60x60 nm contacts which "pin down" the center of the cavity so that it remains flat (Fig.1). The 2DEG is assumed to exist in a thin layer of GaAs. All conducting electrons are assumed to be at the Fermi energy, which is 0.5t (2.85 meV) , where t = $\hbar^2 / 2M^* a^2$, the hopping energy of the mesh used in the numerical calculations. Such a device could readily be fabricated

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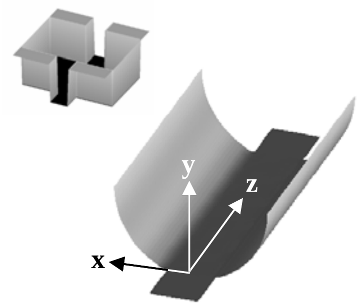

Fig.1 A cylindrical quantum cavity with its potential V_{edge} (top left).

using known techniques [3] where a thin-film heterostructure is grown atop a sacrificial substrate. As the sacrificial layer is etched away underneath, the forces at the heterojunction cause the film to bend toward the layer under tensile strain.

The confinement method for quantum-mechanical modeling of curved two-dimensional electron gases (2DEGs) involves the introduction of an infinitely high, infinitely narrow one-dimensional (1D) potential well into the three-dimensional (3D) Schrödinger equation, effectively trapping the electrons onto the 2D surface of interest. This surface is parameterized using coordinates q_1 and q_2, while q_3 is defined along the normal to the surface at all points. Using separation of variables, $\Psi(q_1,q_2,q_3,t)$ is rewritten as $\psi(q_1,q_2,t)\Psi_N(q_3,t)$ where Ψ_N is assumed to be the ground state. In the case of a cylindrical surface, like the one we are interested in (Fig. 1), ψ obeys the time-independent Schrödinger equation

$$\frac{\hat{p}^2}{2M^*}\psi + V_{edge}\psi + V_g\psi = E\psi \ .$$

The Schrödinger equation for a cylindrical cavity[1] is identical to the equation for a flat cavity [confinement potential V_{edge} (Fig. 1) defines the lateral edges of the cavity], with the addition of an attractive potential $V_g = -\hbar^2/8M^*R_C^2$ of purely geometric origin, where R_C is the local radius of curvature.[2] The presence of a magnetic field can be accounted for through the use of Peierls' phase factor [8], $\psi = \psi_{B=0}\exp\{-i\Phi_\perp/\Phi_0\}$. The resultant 2D Schrödinger equation is discretized, and Usuki's transfer matrix method [9] is employed to calculate conductance and probability density.Baseline calculations were made for a flat cavity in a magnetic field, and the first resonance feature was observed at 0.08 Tesla. Upon curving

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[1] In general, as elaborated by daCosta [1], the kinetic energy operator bears explicit dependence on the components of the metric tensor inherited from the 3D metric, but reduces to the common form given above for a cylindrical surface.

[2] For our tethered cylinder, there is formally a singularity in the equation of motion due to the infinite radius of curvature at the junction where the curved and flat portions meet. However, in a real device this will not be the case, so as an approximation this singularity is ignored.

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the cavity, both the shape of the valley and the magnetic field required to generate it were altered, as shown in Fig. 2.

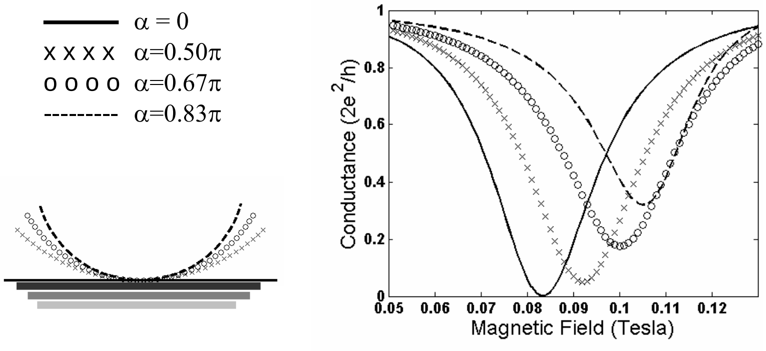

Fig.2 Conductance valley for four different curvatures (right), partially explained by the shrinking footprint (left) and therefore reduced flux through the cavity.

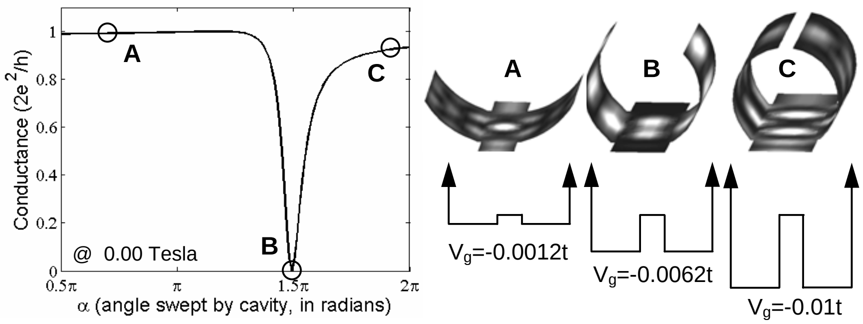

Fig.3 Curvature induces strong resonance even in the absence of a magnetic field. This effect is attributed to the geometric potential.

At low curvature, the shift of this antiresonance is largely attributable to reduced magnetic flux through the cavity. At higher curvatures, however, this correspondence breaks down as other effects become more prominent. Moreover, there is an increase in the minimum of the valley that is not explained by a simple reduction in total flux. With increasing curvature, B⊥ becomes increasingly nonuniform, remaining strong in the center of the cavity while attenuating at the lateral edges, which become more and more parallel to the magnetic field. This causes flattening of the valley. Second, a geometric potential is induced in the wings of the cavity, altering the wavefunction and its ability to couple to the contacts. This is the reason for the leftward shift.

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The most exciting result of this study is the introduction of resonant states by curvature alone, in the absence of a magnetic field (Fig.3). The geometric potential, even though relatively small at these dimensions (approximately E_F /80), was sufficient to produce a well-defined antiresonance. The conductance antiresonance (left panel of Fig. 3) corresponds to a minimum of transmission between leads: in panel B on the right-hand-side, the probability density in part of the cavity connecting to the leads is zero, and this particular wavefunction is an eigenfunction for the closed cavity problem (i.e., leads removed). This effective decoupling from the leads is known [7] to correspond to conductance minima in flat cavities.

The cumulative effects of curvature (the geometric potential, the non-uniform $B_\perp$, and the decrease in total flux) result in a quantum cavity whose conductance can be extremely sensitive to its shape. This suggests possible application as a minute force-detecting MEMS component, or as a sensor for chemical agents that can affect the curvature-inducing strain.

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