Bound States in the Vortex Core

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Abstract

The quasiparticle excitation spectrum of isolated vortices in clean layered d-wave superconductors is calculated. A large peak in the density of states in the "pancake" vortex core is found, in an agreement with the recent experimental data for high-temperature superconductors.

1 Introduction

The vortex core in classical type-II superconductors can be treated as the normal metal area with radius of the order of the coherence length $\xi(0) \sim 10\,\mathrm{nm}$.[1, 2] The spectrum of the bound states near the Fermi surface, formed by the constructive interference between the incident and the Andreev reflected quasiparticles,[3] is quasicontinuous (gapless). High-temperature superconductors (HTS) are layered, having a cylindrical Fermi surface, the superconductivity is of the strong and d-wave coupling type, and the vortex core radius is much smaller, $\xi(0) \sim 1\mathrm{nm}$.

For a two-dimensional (2D) vortex and s-pairing, Rainer *et al.* have shown, using the Andreev quasiclassical theory in the analytical, and Eilenberger's in the numerical part of their study, that bound states exist in the vortex core.[4] Similar conclusions have also been obtained by Maki and coworkers for d-pairing, within the Bogoliubov-de Gennes approach.[5]

In this paper, the Eilenberger quasiclassical equations [6], in the case of both s and d-pairing, are solved analytically, within the model considering the spatial variation of the order parameter in the vortex core as for a normal metal cylinder of radius $r_c \sim \xi$. A crucial difference is found in the quasiparticle spectra below the bulk energy gap, between the classical superconductors with spherical Fermi surface, s-pairing, large ξ, and HTS with cylindrical Fermi surface, d-pairing, small ξ. Our results confirm that the quasiparticle density of states (DOS) has one large maximum, recently observed in YBCO by scanning tunneling microscopy.[7]

2 Model of the Vortex Core

An efficient method for calculating local spectral properties is the quasiclassical theory of superconductivity, which gives the Eilenberger equations

$$\left[2\hbar\omega_n + \hbar\mathbf{v}\cdot\left(\nabla - \imath\frac{2e}{\hbar c}\mathbf{A}\right)\right] f = 2\Delta g, \quad \left[2\hbar\omega_n - \hbar\mathbf{v}\cdot\left(\nabla + \imath\frac{2e}{\hbar c}\mathbf{A}\right)\right] f^\dagger = 2\Delta^* g, \tag{1}$$

$$\hbar\mathbf{v}\cdot\nabla g = \Delta^* f - f^\dagger\Delta. \tag{2}$$

Here $g = g_{\downarrow\downarrow}(\mathbf{r}, \mathbf{v}, \omega_n)$ and $f = f_{\downarrow\uparrow}(\mathbf{r}, \mathbf{v}, \omega_n)$ represent the normal and the anomalous Green function respectively, $\Delta = \Delta(\mathbf{r}, \mathbf{v})$ is the gap function, $\omega_n = \pi k_B T(2n+1)$, $n = 0, \pm 1, \pm 2...$, are the Matsubara's frequencies, and $\mathbf{v}$ is the Fermi velocity vector. The function $f^\dagger$ is defined by $f^\dagger_{\uparrow\downarrow}(\mathbf{r}, \mathbf{v}, \omega_n) = f^*_{\downarrow\uparrow}(\mathbf{r}, -\mathbf{v}, \omega_n)$. Δ and f are connected by the self-consistency equation.

For a homogeneous and isotropic superconductor, solutions of the Eilenberger equations are

$$\langle f \rangle = \frac{\Delta}{\varepsilon_n}, \quad \langle f^\dagger \rangle = \frac{\Delta^*}{\varepsilon_n}, \quad \langle g \rangle = \frac{\hbar\omega_n}{\varepsilon_n} \qquad \left(\varepsilon_n^2 = |\Delta|^2 + (\hbar\omega_n)^2\right). \tag{3}$$

In the cylindrical coordinates (r, φ, z), with the origin situated on the vortex axis, the vortex magnetic field is $\mathbf{h} = h\mathbf{e}_z$. At distances $r \sim \xi \ll \lambda$ from the vortex axis, h is approximately constant, and the gauge can be chosen in the form $\mathbf{A} = (\mathbf{h} \times \mathbf{r})/2$. Taking the same vortex gap function as for a normal metal cylinder embedded in a superconductor

$$\Delta = \Delta(r, \theta) e^{-\imath\varphi}\ , \ \Delta(r, \theta) = \begin{cases} 0, & r \le r_c \\ \Delta(\theta), & r > r_c \end{cases} \quad . \tag{4}$$

Here, r_c is the vortex core radius, and θ is the polar angle in $\mathbf{k}$-space. For d-pairing $\Delta(\theta) = \Delta_0 \cos 2\theta$, and for s-pairing $\Delta(\theta) = \Delta_0$. The gauge in Eq. (4) is due to the flux quantization.

For a pancake vortex in (r, φ) plane, denoting the coordinate along $\mathbf{v}$ by s, and along $\mathbf{h} \times \mathbf{v}$ by p (Fig.1.), in the gauge with real gap, Eqs. (1) and (2) can be rewritten in the form

$$\left[2\hbar\omega_n + \hbar \mathrm{v} \left(\frac{\partial}{\partial s} + \imath\frac{p}{l_H^2} + \imath\frac{p}{r^2}\right)\right] f = 2\Delta(r, \theta) g, \tag{5}$$

$$\left[2\hbar\omega_n - \hbar \mathrm{v} \left(\frac{\partial}{\partial s} - \imath\frac{p}{l_H^2} - \imath\frac{p}{r^2}\right)\right] f^\dagger = 2\Delta(r, \theta) g, \tag{6}$$

$$\hbar \mathrm{v} \frac{\partial}{\partial s} g = \Delta(r, \theta)(f - f^\dagger), \tag{7}$$

where $r^2 = p^2 + s^2$ and $l_H^2 = \hbar c/eh$.

For a normal metal cylinder and zero magnetic field, Eqs. (5)-(7) with $p = 0$, the solution is of the form

$$f = \sum_i f_i(p) e^{\kappa_i s}, \quad g = \sum_i g_i(p) e^{\kappa_i s}. \tag{8}$$

For $r \le r_c$, with $\kappa_0 = 2\omega_n/\mathrm{v}$,

$$f = F e^{-\kappa_0 s}, \quad g = G. \tag{9}$$

For $r > r_c$ and $\kappa = 2\varepsilon_n/\hbar\mathrm{v}$,

$$f = \langle f \rangle + \Phi_1 e^{-\kappa s}, \quad g = \langle g \rangle + \Gamma_1 e^{-\kappa s}, \quad \text{for } s > 0, \tag{10}$$

$$f = \langle f \rangle + \Phi_2 e^{\kappa s}, \quad g = \langle g \rangle + \Gamma_2 e^{\kappa s}, \quad \text{for } s \le 0. \tag{11}$$

Eqs. (5)-(7) imply

$$\frac{\Phi_1}{\Gamma_1} = \frac{\Delta(\theta)}{\hbar\omega_n - \varepsilon_n}, \quad \frac{\Phi_2}{\Gamma_2} = \frac{\Delta(\theta)}{\hbar\omega_n + \varepsilon_n}. \tag{12}$$

Using the continuity condition for f and g at $\pm s_0 = \pm\sqrt{r_c^2 - p^2}$, for $r < r_c$ the normal Green function is

$$G = \frac{\hbar\omega_n \cosh(\kappa_0 s_0) + \varepsilon_n \sinh(\kappa_0 s_0)}{\hbar\omega_n \sinh(\kappa_0 s_0) + \varepsilon_n \cosh(\kappa_0 s_0)}. \tag{13}$$

For a vortex, approximating p/r^2 by p/r_c^2, the solution of Eq. (5)-(7) can be obtained from Eq. (13), by changing $\omega_n \to {\omega_n}'$,

$$G \approx \frac{\hbar\omega_n' \cosh(\kappa_0' s_0) + \varepsilon_n' \sinh(\kappa_0' s_0)}{\hbar\omega_n' \sinh(\kappa_0' s_0) + \varepsilon_n' \cosh(\kappa_0' s_0)}, \tag{14}$$

where

$${\omega_n}' = \omega_n + \imath\frac{p\mathrm{v}}{2}\left(\frac{1}{r_c^2} + \frac{1}{l_H^2}\right), \tag{15}$$

and $\kappa_0' = 2\omega_n'/\mathrm{v}$. In this case, the magnetic flux quantization leads to [1, 5]

$$p_i = \left(i + \frac{1}{2}\right)\frac{\hbar}{m\mathrm{v}}, \quad i = 0, \pm1, \pm2, ... \tag{16}$$

Since for an isolated vortex $l_H \gg r_c$, the direct influence of the field can be neglected, and the only relevant contribution is due to the screening supercurrent flow, $\imath p\mathrm{v}/2r_c^2$ term in Eq. (15).

3 Bound States

Performing an analytical continuation of G by $\hbar\omega_n \to -\imath E + \eta$, E being the quasiparticle energy with respect to the Fermi level, the retarded propagator $g^R(E,p,\theta)$ is obtained. In terms of reduced variables $E/\Delta_0 \to E$, $\sqrt{\Delta^2(\theta) - E^2}/\Delta_0 \to \varepsilon$, $\sqrt{E^2 - \Delta^2(\theta)}/\Delta_0 \to e$, $p/r_c \to p$, $2s_0/\pi\xi_0 \to s_0$, $\xi_0 = \hbar\mathrm{v}/\pi\Delta_0$ being the BCS coherence length, angle resolved partial DOS (PDOS) is obtained from $N(E,p,\theta) = \mathrm{Re} g^R(E,p,\theta)$.

For the normal metal cylinder

$$\begin{aligned} N(E,p,\theta)/N(0) &= \Theta\left(e^2\right)\frac{|E|e}{e^2\cos^2(Es_0) + E^2\sin^2(Es_0)} + \\ &+ \Theta\left(\varepsilon^2\right)\frac{\pi|\Delta(\theta)|}{\Delta_0}\delta\left(E\sin(Es_0) - \varepsilon\cos(Es_0)\right), \end{aligned} \tag{17}$$

where δ is the Dirac function, Θ is the step-function, and $N(0) = m/2\pi\hbar^2$ is the normal metal density of states at the Fermi surface for one spin orientation. For s-wave pairing, PDOS does not depend on θ, while for d-wave pairing, averaging over the cylindrical Fermi surface leads to

$$\begin{aligned} N(E,p)/N(0) &= \frac{1}{2\pi}\int_0^{2\pi} N(E,p,\theta)/N(0)\, d\theta = \\ &= \frac{2}{\pi}\int_{\sqrt{\max\{0,E^2-1\}}}^{|E|} \frac{|E|e^2\, de}{\sqrt{E^2 - e^2}\sqrt{1 - E^2 + e^2}\left(e^2\cos^2(Es_0) + E^2\sin^2(Es_0)\right)} + \\ &+ \frac{\left(E\tan(Es_0) + |E\tan(Es_0)|\right)}{\sqrt{\cos^2(Es_0) - E^2}}\Theta\left(\cos^2(Es_0) - E^2\right). \end{aligned} \tag{18}$$

Finally, after spatial averaging over the cylinder area πr_c^2, DOS is

$$N(E) = \frac{4}{\pi}\int_0^1 N(E,p)\sqrt{1 - p^2}\, dp. \tag{19}$$

For the vortex, in Eqs. (17) and (18), $E \to E + E_0 \operatorname{sign} E$, $E_0 = \hbar|p_i|\mathrm{v}/2\Delta_0 r_c^2$, Eq. (15), with p_i from Eq. (16), and $\sum_{p_i}$ instead of integration in Eq. (19). Here, signs of p and E are connected, because the magnetic field causes a difference in propagation of particles and holes.

For small radius vortices in HTS, $\xi_0 m\mathrm{v}/\hbar \sim 1$ (~ 10 in classical superconductors), only one trajectory through the vortex core is allowed, with $p = p_0$, Eq. (16). Taking for YBCO $r_c = \xi_0$, $p_0 = 1/3$, $\Delta_0/E_F = 0.424$, only one peak in DOS in the vortex core around $E/\Delta_0 \approx 0.3$ is obtained (Fig. 2). For comparison, DOS of normal metal cylinder embedded in the same superconductor and with the same radius $r_c = \xi_0$, but in the zero magnetic field, is shown. In this case, a large energy gap is found in DOS, due to formation of lowest bound state at high energy, of the order of Δ_0. This is not the case in classical superconductors, where $\Delta_0/E_F \ll 1$.

Fig.1. Trajectory passing at distance p from the vortex center.

Fig.2. Quasiparticle DOS in the vortex core (solid curve), and in the normal metal cylinder (dashed curve), for clean layered d-wave superconductor. $r_c/\xi_0 = 1$, $\Delta_0/E_F = 0.424$.

In conclusion, cylindrical Fermi surface, d-wave pairing and small ξ_0, large $\Delta_0/E_F \sim 0.1$, make DOS of a normal metal cylinder embedded in HTS and a pancake vortex different from DOS of a normal cylinder and a vortex in classical superconductors. Since the Andreev bound states can transport charge currents, unlike the bound states in a potential well, supercurrents can flow through the vortex without losses, strongly influencing its dynamics. This could be very important for transport properties of HTS, especially for understanding the unusual magnetic-field dependence of the electrothermal conductivity, which was observed experimentally and awaits explanation.[8]

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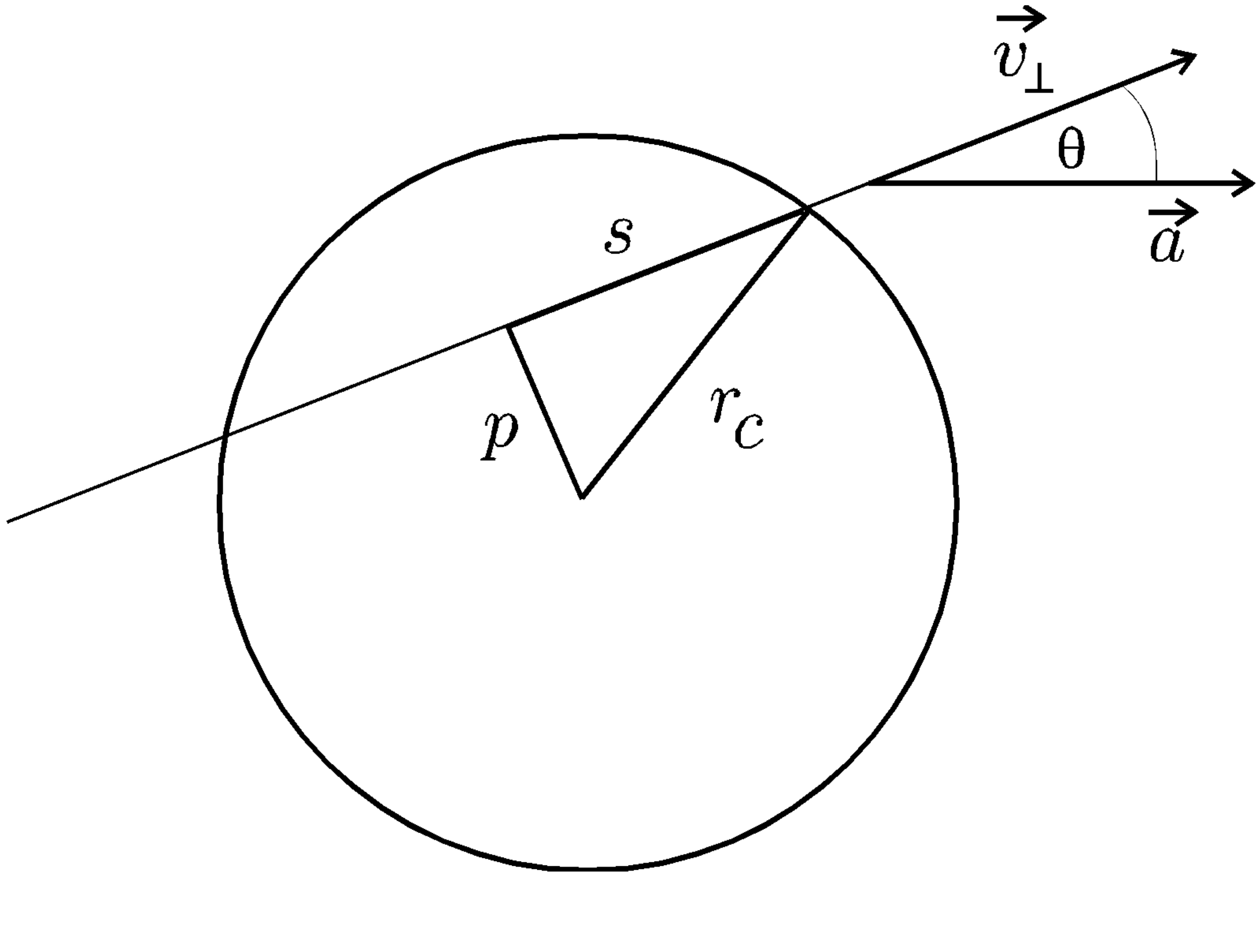

Figure 1

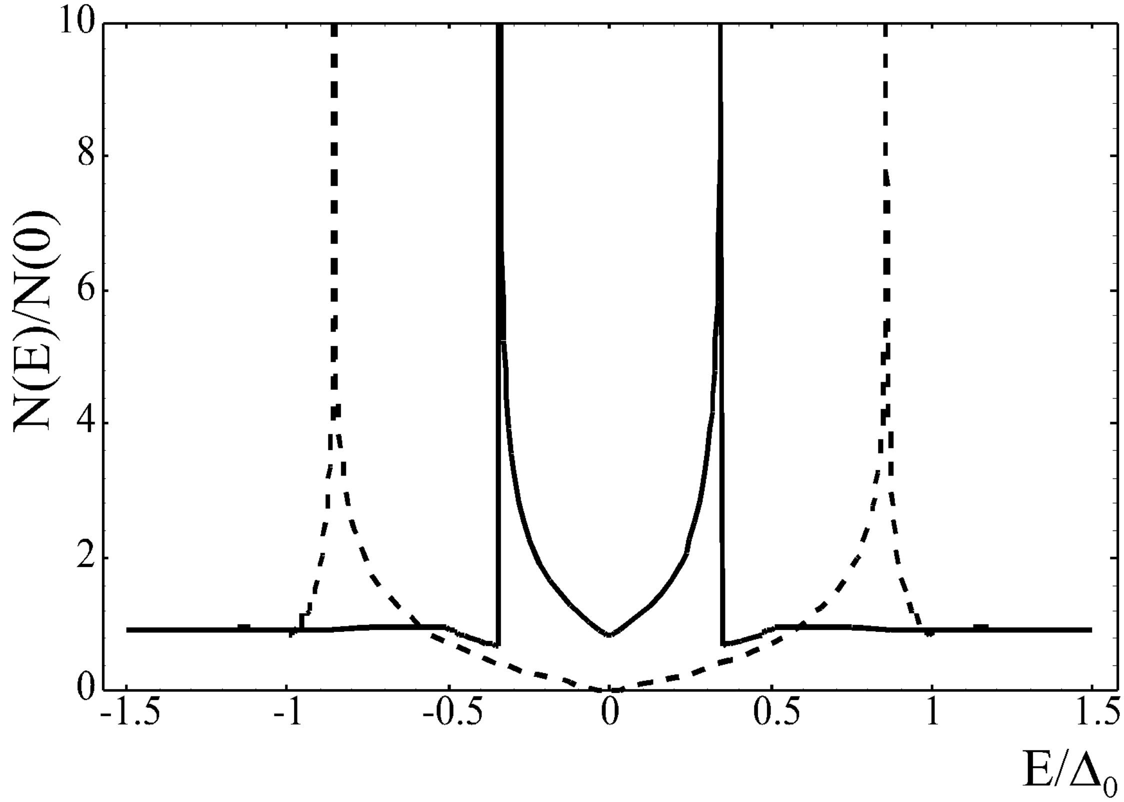

Figure 2