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Quantum waves—the proper basis for low dissipation quantum computing

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Abstract

The promise of more effective computation led to quantum computers, which are supposed to surpass binary, digital computers. This is an outgrowth of the search for dissipation-free computation, in which the bit state is switched 'adiabatically.' Implementations in which the two local minima are at different energy levels are unstable. Using super-symmetric quantum mechanics, we show that natural transitions occur between degenerate eigenstates. Then we can write, quite generally, that the qubit should satisfy the Hamiltonian $H = k^2 > 0$, and we associate k with the 'momentum' in Hilbert space. That is, *the fundamental structure of the quantum qubit should be that of a quantum plane wave.*

1. Introduction

In recent years, quantum computing and quantum information theory have rapidly developed. The structure of quantum computation has centered around the use of a *qubit* [1], which is a generalization of a simple binary state. Typically, we think of a *binary* bit being in one of two states, which we traditionally describe as a '0' or a '1' state. In logic circuits, for example, the '1' state may be taken to be the state with a high voltage, while the '0' state is represented by a state with low (or zero) voltage. It is possible to think of this assembly of bits as composing a Hilbert space, in which each coordinate axis corresponds to one bit. In this case, the projection of the system onto that axis can take only the values '1' or '0', corresponding to the value of that bit. Thus, the system vector cannot point to just any point in the Hilbert space, but only to that set of discrete points for which each bit has a value of '0' or '1'. In *quantum computing*, the '0' and '1' values are taken as separate coordinates in the Hilbert space. This allows us to think of the qubit as a 'two-state system.' For the 'qubit,' or quantum bit, the major change is to represent the coefficients as arising from a quantum wave function. However, the projection onto the two coordinates is no longer limited to discrete values. Rather, *a*

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continuous range of projection onto these states is now possible. Hence, we have effectively made the states *analog* states—as they are no longer digital.

Recently, one suggestion for a semiconductor qubit actually utilizes the interference of quantum waves contained within two parallel waveguides [2]. Here, control of the active length between two slits, through which the wave (in the two adjacent guides) can interact, generates the proposed qubit. Although these authors have used wave packets, a full plane wave approach is also possible [3]. More importantly, Bertoni et al. have also shown how two such waveguide qubits can establish entanglement [4]. In this paper, we show that a natural implementation of the qubit is via quantum plane waves, in particular laterally-confined propagating waves of the type proposed in these recent papers.

2. Quantum mechanics of the qubit

We may express the two coordinates of the qubit in terms of a wave function in an appropriate representation, according to ($\alpha^2 + \beta^2 = 1$)

$$|\psi\rangle = \alpha\, e^{i\gamma_1}|\psi_0\rangle + \beta\, e^{i\gamma_2}|\psi_1\rangle \tag{1}$$

so that the wave function can in general be written as

$$|\psi\rangle = e^{i\gamma_1}(\alpha|0\rangle + e^{i\xi}\beta|1\rangle) \tag{2}$$

where

$$\xi = \gamma_2 - \gamma_1 \tag{3}$$

Hence, we can define the density matrix

$$\rho = \begin{bmatrix} \alpha^2 & \alpha\beta\, e^{-i\xi} \\ \alpha\beta\, e^{i\xi} & \beta^2 \end{bmatrix} \tag{4}$$

which is idempotent, and represents the pure state (2) for the qubit. This is a standard approach for a two-level system, and the density matrix represents a projection operator onto the state $|\psi\rangle$. In general, this density matrix can be written as

$$\rho = \frac{1}{2}(\mathbf{I} + \mathbf{P}\cdot\boldsymbol{\sigma}) \tag{5}$$

where $\mathbf{I}$ is a unit matrix and $\mathbf{P}$ is the polarization matrix

$$P_x = 2\alpha\beta\cos(\xi), \quad P_y = 2\alpha\beta\sin(\xi), \quad P_z = \alpha^2 - \beta^2, \tag{6}$$

and whose direction defines the 'spin' of the two state system.

For such a system, if two levels are not degenerate, the Hamiltonian in general has a term of the form $\mathbf{Q}\cdot\boldsymbol{\sigma}$, in which $\mathbf{Q}$ represents the precession 'frequency' of the polarization $\mathbf{P}$. The precession frequency is given by the energy difference between the two levels. The angle ξ of the qubit is a rotation in the (x, y)-plane and the precession frequency $\mathbf{Q}$ is usually aligned in the z-direction. In this sense, the precession term corresponds to the spin energy in a magnetic field in this two-level model. In fact, $\mathbf{Q}$ is normally associated with an effective magnetic field.

However, the problem is essentially that of a spin-1/2 electron in a magnetic field, which can be reduced to a 1D harmonic oscillator problem. This is the proto-typical problem, most easily handled with super-symmetric quantum mechanics (SUSY-QM) [5], in that we can associate a confining potential $W^2(x)$ with each of the two energy 'minima,' but with the two separated by the spin. We can combine the boson transformation (energy levels in a single well) with fermion transformations (the iso-spin) into a single group (the 'supergroup'). The principle is that after rescaling the potential, the lowest energy level has zero energy in a single potential well, for which the Hamiltonian operator is given by ($\hbar = 1$, $m = 1/2$) [6]

$$H^{(-)} = -\frac{d^2}{dx^2} + W^2(x) + W'(x) = A^+ A \tag{7}$$

where

$$W(x) = \frac{d}{dx}\ln\left[\psi_0^{(-)}(x)\right] \rightarrow -\frac{1}{2}x \tag{8}$$

is termed the 'superpotential' (ψ_0 is the ground state wave function, in this case just the simple Gaussian of the lowest harmonic oscillator level) and the 'supercharge' operator A annihilates the ground state ($[A, A^+] = 2W' \rightarrow 1$ for the simple case here). The partner Hamiltonian (for, e.g. the second set of levels which arise from the '1' state) is given by

$$H^{(+)} = -\frac{d^2}{dx^2} + W^2(x) - W'(x) = AA^+ \tag{9}$$

The important point is that the lowest eigenvalue of this latter Hamiltonian is degenerate with the first eigenvalue of Eq. (7). That is, the eigenvalues of the total system are all doubly degenerate except for the lowest one of Eq. (7).

If we were to use two energy levels within a single harmonic oscillator, the excitation operator (boson creation operator) does not commute with the Hamiltonian, and significant energy exchange must occur when 'mixing' the two states to achieve entanglement. On the other hand, the operators A and A^+ move horizontally between the two ladders of states, as shown in Fig. 1. Moreover, the supercharge (and its conjugate) commute with the Hamiltonians (8) and (9), so that these lateral transitions are made without changing the energy of the system.

As a consequence, it is desirable to have the two states, which represent the two choices for the bit or qubit, degenerate in energy, as indicated in Fig. 1 by the lateral transitions corresponding to the operators A and A^+. These two operators commute with the two Hamiltonians, so that the transitions can truly be made in an adiabatic manner, and entanglement of these two states can be induced by these operators. Hence, choices such as magnetic domain orientation and electron spin are natural choices for such a qubit. On the other hand, when the two states are at the same energy level, then there is no precession of this isospin polarization in Eq. (5), and it follows that the Hamiltonian of the two-level system can be written as [7]

$$H = \begin{bmatrix} C_0 & 0 \\ 0 & C_0 \end{bmatrix}, \quad [H, \rho] = 0. \tag{10}$$

In keeping with Eq. (10), the simplest (and at the same time most general) linear system will have a

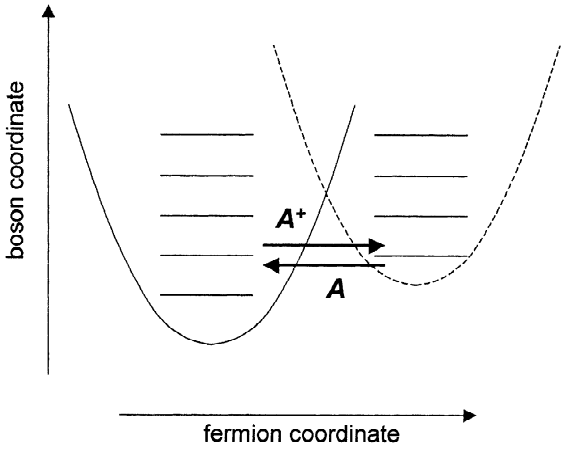

Fig. 1. The coordinates of the basic Hamiltonian and its partner. The left-hand potential (solid) represents that of $H^{(-)}$ while the right-hand potential (dashed) represents that of $H^{(+)}$. The supercharge, and its conjugate, move laterally from one ladder of levels to the other.

quadratic Hamiltonian. This Hamiltonian is Hermitian, so that C_0 is real (and positive, since the energies are positive). We can write, quite generally, that the qubit should satisfy a Hamiltonian whose expectation value is simply

$$\hat{H} = C_0 = k^2 \tag{11}$$

In choosing the value k, we are guided by the Schrödinger equation itself, so that we associate k with the 'momentum' for a mode (which may be in a confined system). The Hamiltonian is now isomorphic to that for a free (but perhaps confined) *wave* propagating in space. That is, the fundamental structure of the quantum qubit should be that of a quantum wave.

The use of semiconductors and quantum waves offers a wave velocity that is two orders of magnitude larger than acoustic velocities and three orders of magnitude smaller than radio wave velocities. At the same time, frequencies into the far infrared are possible. Typical velocities correlate with the Fermi energy, which is typically on the order of 10–20 meV in quasi-two dimensional electron gases. This corresponds to approximately 10–20 THz. At the same time, the crucial wavelength is the Fermi wavelength, which is 30–50 nm. With wave velocities on the order of 10^7 cm/s, and phase coherent times on the order of 10^{-10} s, this means that coherent lengths of the order of 10 μm will be possible at very low temperatures. As a result, it is clear that quantum wave processing can be achieved with nano-technology, with individual feature sizes on the 30–50 nm scale and with complete processing structures smaller than this 10 μm scale. This coherence length is some 200 wavelengths, so that relatively complicated structures can be created using electron-beam lithography and nanofabrication in GaAs/AlGaAs structures [8,9]. Hence, *nano-lithography, coupled with high mobility heterostructures, offers promise for quantum wave computation involving hundreds of qubits.*

3. Summary

Quantum computation is really a highly parallel analog processing that is provided by the array of qubits. As a result, the natural basis for the qubit is that of a quantum plane wave, freely propagating in the medium. These quantum waves have very promising attributes, and they can be incorporated into structures using nanotechnology. Use of high mobility heterostructures, such as GaAs/AlGaAs, promises that phase coherent structures of several microns in extent can be conceived and demonstrated. We feel that the future can be quite bright for quantum wave processing as a meaningful implementation of quantum computing.

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