Problem 1. (25%)  

The three solid shafts have the same diameters (3/4 inch) and the same modulus of rigidity (11x10^6 psi). All shaft support locations can be considered as bearings, except for location D, which is completely constrained. Known torques are applied at ends A and B as shown.

(a) What is the magnitude of the maximum shear stress in shaft AB?
(b) What is the magnitude of the maximum normal stress in shaft AB?
(c) What is the rotation of gear C (degrees)?

\[ \tau_{\text{max}} = \frac{100 \text{ lb-in}}{\frac{5}{3} \text{ in}^3} = 1.207 \text{ ksi} \]

\[ J = 3.1 \times 10^{-2} \text{ in}^4 \]

\[ \theta = \frac{180 \times 1000 \text{ lb} \cdot \text{in} \cdot 3.12 \text{ in}}{\frac{5}{3} \times 11 \times 10^6 \text{ psi}} = 6.035^0 \]
Problem 2. (25%)
Two blocks of rubber, each of width \( w = 100 \text{ mm} \) are bonded to rigid supports and the movable plate \( AB \). The thicknesses of the top and bottom blocks are 25 mm and 40 mm, respectively, but the modulus of rigidity is the same, 7.5 MPa. A horizontal centric force \( P = 26 \text{ kN} \) is applied to the plate \( AB \).

(a) Determine the horizontal displacement \( \delta \) of the plate \( AB \).

(b) Determine the average shear stress and shear strain in each of the rubber blocks.

(c) Determine the magnitudes of the forces carried by the upper and lower rigid supports.

(Hint: Note that this problem is statically indeterminate.)

\[
P = P_{CD} + P_{EF} \quad (1)
\]

\[
\delta = \gamma_{CD} \cdot h_c = \gamma_{EF} \cdot h_E
\]

\[
\begin{align*}
P_{CD} &= G \cdot \gamma_{CD} \cdot A \quad &\Rightarrow & \quad P_{CD} = \frac{P_{EF}}{\delta_{CD}} = \frac{P_{EF}}{\delta_{EF}} \\
P_{EF} &= G \cdot \gamma_{EF} \cdot A
\end{align*}
\]

\[
\frac{P_{CD} \cdot h_c}{\delta} = \frac{P_{EF} \cdot h_E}{\delta} \quad \text{or} \quad \frac{P_{CD}}{h_c} \cdot h_c = \frac{P_{EF}}{h_E} \cdot h_E \quad (2)
\]

\[
P_{CD} = P - P_{EF} \Rightarrow P_{EF} = P \frac{h_c}{h_c + h_E} \quad \text{and} \quad P_{CD} = P \frac{h_E}{h_c + h_E}
\]

\( c) \quad P_{EF} = 10 \text{ kN} \quad \text{and} \quad P_{CD} = 16 \text{ kN} \)

\( b) \quad \tau_{CD} = \frac{P_{CD}}{A} = \frac{16000 \text{ N}}{100 \times 180 \times 10^{-6} \text{ m}^2} = 889.9 \text{ kPa} \quad \gamma_{CD} = \frac{\tau_{CD}}{G} = 0.118
\]

\[
\tau_{EF} = \frac{P_{EF}}{A} = \frac{10000 \text{ N}}{100 \times 180 \times 10^{-6} \text{ m}^2} = 555.6 \text{ kPa} \quad \gamma_{EF} = \frac{\tau_{EF}}{G} = 0.034
\]

\( b) \quad \delta = \gamma_{CD} \cdot h_c = \frac{\gamma_{CD}}{G} \cdot h_c = 2.96 \text{ mm} \)
Problem 3. (25%)

A rigidly-supported, unreinforced concrete pier of width \( w \) and depth \( d \) must carry the concentrated load \( F \) somewhere along the line \( ACB \).

(a) At what maximum distance \( s \), from the center \( C \) of the pier, can the load \( F \) be placed so that only compressive stresses act on the plane \( P \) near the base of the pier. Express \( s \) as a function of \( w \), the width of the pier. (Hint: The answer does not depend on the magnitude of \( F \)).

(b) For the particular case of \( s = 0.10 \, \text{m} \), determine the normal stress acting at point \( E' \) on the plane \( P \), near the base of the pier. Use: \( w = 1 \, \text{m} \), \( d = 0.3 \, \text{m} \), and \( F = 1000 \, \text{kN} \).

\[
\sigma = \frac{F}{A} - \frac{My}{I} = F \left( \frac{1}{A} - \frac{s y}{I} \right) \quad \text{where} \quad I = \frac{1}{12} dw^3
\]

\(F < 0 \quad \sigma < 0 \quad \frac{1}{A} - \frac{s y}{I} > 0
\]

Let \( y = \frac{w}{2} \)

\[
s \leq A \frac{w}{2} = \frac{1}{12} A w \frac{w}{2} = \frac{w}{6} \quad \triangle
\]

\(6) \quad \sigma_E = -1000 \cdot 10^3 N \left( \frac{1}{0.3 \cdot 1m^2} + \frac{0.1m \cdot 0.5m}{12 \cdot 0.3 \cdot 1^3 m^4} \right)
\]

\[
= -10^6 \text{Pa} \left( \frac{1}{0.3} + 4 \cdot 0.5 \right) = -5.33 \cdot 10^6 \text{Pa} \quad \triangle
\]
Problem 4. (25%) 

The simply-supported beam is loaded by a concentrated force of 20 kN. The hat-shaped cross section shows horizontal flanges which are 4 mm thick and vertical webs which are 6 mm thick.

(a) Calculate the support forces at each end of the beam.

(b) At location $A-B$, determine the flexural (bending) stresses at the top and bottom of the cross section.

(c) At location $A-B$, calculate the shear stress for points on the midsurface, i.e., 30 mm from the top and the bottom.

\[
\begin{align*}
\text{2000 mm} & \quad \text{20 kN} \\
A & \quad B \\
1500 \text{ mm} & \quad \text{3000 mm}
\end{align*}
\]

\[ I = \frac{1}{12} (0.04 \times 0.06^3 - 0.028 \times 0.05^2) \text{m}^4 = 3.9 \times 10^{-7} \text{m}^4 \]

\[ M_{BA} = F_c \times 1.5\text{m} = 6.7 \times 10^{-3} \times 1.5 \text{N} \cdot \text{m} = 10.05 \text{ N} \cdot \text{m} \]

\[ \varphi = 0.04 \times 0.08 \times 0.015 \text{m}^3 - 0.028 \times 0.026 \times 0.013 \text{m}^3 = 8.5 \times 10^{-6} \text{m} \]

\[ V = F_c = 6.7 \times 10^2 \text{N} \]

\[ \varepsilon = \frac{V \cdot \varphi}{I \cdot 2t}, \text{ where } t = 0.006 \text{m} \]

\[ \varepsilon = 12.2 \text{ MPa}. \]