Two forces $P_1$ and $P_2$, of magnitude $P_1 = 15 \text{kN}$ and $P_2 = 18 \text{kN}$, are applied as shown to the end $A$ of bar $AB$, which is welded to a cylindrical member $BD$ of radius $c = 20 \text{ mm}$ (Fig. 8.21). Knowing that the distance from $A$ to the axis of member $BD$ is $a = 50 \text{ mm}$ and assuming that all stresses remain below the proportional limit of the material, determine (a) the normal and shearing stresses at point $K$ of the transverse section of member $BD$ located at a distance $b = 60 \text{ mm}$ from end $B$, (b) the principal axes and principal stresses at $K$, (c) the maximum shearing stress at $K$. 

**Step 0: Forces & Moments**

**Due to $P_1$**

- $P_1 = -15 \text{kN}$
- $M_1 = P_1 \cdot 0.05 = 750 \text{ Nm}$ (positive $y$ direction)
- $V_1 = 0$
- $T_1 = 0$

**Due to $P_2$**

- $P_2 = 0$
- $M_2 = P_2 \cdot 0.06 = 1080 \text{ Nm}$ (positive $z$ direction)
- $V_2 = 18 \text{kN}$
- $T_2 = 18 \cdot 0.05 = 750 \text{ Nm}$ (positive $x$ direction)

$\overline{NA}$ does not pass through $K$.

**Step 2: Resultant moment; considering $\overline{NA}$, etc.**

- $P = -15 \text{kN}$
- $M = M_1 = 750 \text{ Nm}$ (since $\overline{NA}$ due to $M_2$ passes through $K$)
- $V = 18 \text{kN}$ (NA does not pass through $K$)
- $T = 750 \text{ Nm}$

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**Fig. 8.21**

**Fig. 8.22**

**Fig. 8.23**
Step 3: Primary stresses

\[ A = 1.257 \times 10^{-3} \text{ m}^2 \]
\[ I = 125.7 \times 10^{-9} \text{ m}^4 \]
\[ J = 251.3 \times 10^{-9} \text{ m}^4 \]

\[ \tau_{\text{axial}} = \frac{-15600}{A} = -11.9 \text{ MPa} \]
\[ \tau_{\text{bending}} = +7500 \frac{C}{I} = +119.3 \text{ MPa} \]

\[ \varepsilon_{\text{shear}} = \frac{4}{3} \frac{V}{A} = 19.1 \text{ MPa} \]

\[ \tau_{\text{torision}} = \frac{Tr}{J} = -52.5 \text{ MPa} \]
Step 4 Net Stress

\[ \sigma_x = \sigma_{\text{bending}} + \sigma_{\text{axial}} \]
\[ \sigma_x = +107.4 \text{ MPa} \]
\[ \tau_{xy} = \tau_{\text{torque}} + \tau_{\text{shear}} \]
\[ \tau_{xy} = -52.5 \text{ MPa} \]

Mohr Circle

\[ \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \]
\[ \tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}} \]
\[ R = 75.1 \text{ MPa} \]
\[ \sigma_1 = 128.8 \text{ MPa} \]
\[ \sigma_2 = -21.4 \text{ MPa} \]
SAMPLE PROBLEM 8.4

A horizontal 500-lb force acts at point D of crankshaft AB which is held in static equilibrium by a twisting couple T and by reactions at A and B. Knowing that the bearings are self-aligning and exert no couples on the shaft, determine the normal and shearing stresses at points H, J, K, and L located at the ends of the vertical and horizontal diameters of a transverse section located 2.5 in. to the left of bearing B.

SOLUTION

Free Body. Entire Crankshaft. \( A = B = 250 \text{ lb} \)

\[ + \sum M_x = 0: \quad -(500 \text{ lb})(1.8 \text{ in.}) + T = 0 \quad T = 900 \text{ lb \cdot in.} \]

Internal Forces in Transverse Section. We replace the reaction B and the twisting couple T by an equivalent force-couple system at the center C of the transverse section containing H, J, K, and L.

\[ V = B = 250 \text{ lb} \quad T = 900 \text{ lb \cdot in.} \]

\[ M_y = (250 \text{ lb})(2.5 \text{ in.}) = 625 \text{ lb \cdot in.} \]

The geometric properties of the 0.9-in.-diameter section are

\[ A = \pi(0.45 \text{ in.})^2 = 0.636 \text{ in}^2 \quad I = \frac{1}{4}\pi (0.45 \text{ in.})^4 = 32.2 \times 10^{-3} \text{ in}^4 \]

\[ J = \frac{1}{4}\pi (0.45 \text{ in.})^4 = 64.4 \times 10^{-3} \text{ in}^4 \]

Stresses Produced by Twisting Couple T. Using Eq. (3.8), we determine the shearing stresses at points H, J, K, and L and show them in Fig. (a).

\[ \tau = \frac{Tc}{J} = \frac{(900 \text{ lb \cdot in.})(0.45 \text{ in.})}{64.4 \times 10^{-3} \text{ in}^4} = 6290 \text{ psi} \]

Stresses Produced by Shearing Force V. The shearing force V produces no shearing stresses at points J and L. At points H and K we first compute \( Q \) for a semicircle about a vertical diameter and then determine the shearing stress produced by the shear force \( V = 250 \text{ lb} \). These stresses are shown in Fig. (b).

\[ Q = \left( \frac{1}{2} \pi c^2 \right) \left( \frac{4c}{3\pi} \right) = \frac{2}{3} c^3 = \frac{2}{3} (0.45 \text{ in.})^3 = 60.7 \times 10^{-3} \text{ in}^3 \]

\[ \tau = \frac{VQ}{I} = \frac{(250 \text{ lb})(60.7 \times 10^{-3} \text{ in}^3)}{(32.2 \times 10^{-3} \text{ in}^4)(0.9 \text{ in.})} = 524 \text{ psi} \]

Stresses Produced by the Bending Couple \( M_y \). Since the bending couple \( M_y \) acts in a horizontal plane, it produces no stresses at H and K. Using Eq. (4.15), we determine the normal stresses at points J and L and show them in Fig. (c).

\[ \sigma = \frac{|M_y|c}{I} = \frac{(625 \text{ lb \cdot in.})(0.45 \text{ in.})}{32.2 \times 10^{-3} \text{ in}^4} = 8730 \text{ psi} \]

Summary. We add the stresses shown and obtain the total normal and shearing stresses at points H, J, K, and L.
SAMPLE PROBLEM 8.5

Three forces are applied as shown at points A, B, and D of a short steel post. Knowing that the horizontal cross section of the post is a 40 × 140-mm rectangle, determine the principal stresses, principal planes and maximum shearing stress at point H.

SOLUTION

Internal Forces in Section EFG. We replace the three applied forces by an equivalent force-couple system at the center C of the rectangular section EFG. We have

\[ V_x = -30 \text{ kN} \quad P = 50 \text{ kN} \quad V_z = -75 \text{ kN} \]

\[ M_x = (50 \text{ kN})(0.130 \text{ m}) = 6.5 \text{ kN} \cdot \text{m} \]

\[ M_x = 0 \quad M_z = (30 \text{ kN})(0.100 \text{ m}) = 3 \text{ kN} \cdot \text{m} \]

We note that there is no twisting couple about the y axis. The geometric properties of the rectangular section are

\[ A = (0.040 \text{ m})(0.140 \text{ m}) = 5.6 \times 10^{-3} \text{ m}^2 \]

\[ I_x = \frac{1}{12}(0.040 \text{ m})(0.140 \text{ m})^3 = 9.15 \times 10^{-6} \text{ m}^4 \]

\[ I_z = \frac{1}{12}(0.140 \text{ m})(0.040 \text{ m})^3 = 0.747 \times 10^{-6} \text{ m}^4 \]

Normal Stress at H. We note that normal stresses \( \sigma \) are produced by the centric force \( P \) and by the bending couples \( M_x \) and \( M_z \). We determine the sign of each stress by carefully examining the sketch of the force-couple system at C.

\[ \sigma_y = \frac{P}{A} + \frac{M_x a}{I_x} - \frac{M_z b}{I_z} \]

\[ = \frac{50 \text{ kN}}{5.6 \times 10^{-3} \text{ m}^2} + \frac{(3 \text{ kN} \cdot \text{m})(0.020 \text{ m})}{0.747 \times 10^{-6} \text{ m}^4} - \frac{(8.5 \text{ kN} \cdot \text{m})(0.025 \text{ m})}{9.15 \times 10^{-6} \text{ m}^4} \]

\[ \sigma_y = 89.3 \text{ MPa} + 80.3 \text{ MPa} - 23.2 \text{ MPa} \]

\[ \sigma_y = 66.0 \text{ MPa} \]

Shearing Stress at H. Considering first the shearing force \( V_x \), we note that \( Q = 0 \) with respect to the \( z \) axis, since \( H \) is on the edge of the cross section. Thus \( V_x \) produces no shearing stress at \( H \). The shearing force \( V_z \) produces a shearing stress at \( H \) and we write

\[ Q = A \frac{V_z}{I_x} = \frac{(0.040 \text{ m})(0.045 \text{ m})(0.0475 \text{ m})}{9.15 \times 10^{-6} \text{ m}^4}(85.5 \times 10^{-6} \text{ m}^3) \]

\[ \tau_{yz} = \frac{V_z Q}{I_x} = \frac{(75 \text{ kN})(85.5 \times 10^{-6} \text{ m}^3)}{9.15 \times 10^{-6} \text{ m}^4}(0.040 \text{ m}) \]

\[ \tau_{yz} = 17.52 \text{ MPa} \]

Principal Stresses, Principal Planes, and Maximum Shearing Stress at H. We draw Mohr’s circle for the stresses at point H

\[ \tan 2\theta_p = \frac{17.52}{33.0} \quad 2\theta_p = 27.96^\circ \quad \theta_p = 13.98^\circ \]

\[ R = \sqrt{(33.0)^2 + (17.52)^2} = 37.4 \text{ MPa} \]

\[ \tau_{\text{max}} = 37.4 \text{ MPa} \]

\[ \sigma_{\text{max}} = OA = OC + R = 33.0 + 37.4 \]

\[ \sigma_{\text{max}} = 70.4 \text{ MPa} \]

\[ \sigma_{\text{min}} = OB = OC - R = 33.0 - 37.4 \]

\[ \sigma_{\text{min}} = -7.4 \text{ MPa} \]