1. Find the largest rectangle that can fit inside the ellipse:

\[
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1
\]

by posing a one variable optimization problem and solving for the minima analytically.

**Solution and Comments:**

\[
A = 2ab \ldots \text{straightforward}
\]

2. Consider the function \( f(x) = x^3 - x^2 + 2 \). Find and classify the stationary points using analytical methods.

**Solution and Comments:**

\[\text{Maxima} \atop \text{Minima} \atop @ 0 \ at 2/3\]

3. Consider a beam of sectional modulus EI and length L that is pinned at both ends. A load \( P \) is applied at a distance \( L/3 \) from the left end and a load \( P/2 \) is applied at the center. Find the location and value of maximum deflection. (You need to review basic Mechanics of Materials)

**Solution and Comments:**

Note that different deflection equations apply in different sections. The minima occurs in the second section

\[x = 0.473L\]

\[y_{\text{min}} = -0.02826\frac{PL}{EI}\]

4. (Refraction Law of Optics). Let \( p \) and \( q \) be two points on the plane that lie on opposite sides of a horizontal axis. Assume that the speed of light from \( p \) to the horizontal axis is \( v \), and from the horizontal axis to \( q \) is \( w \). Find the fastest path from \( p \) to \( q \) (the path that a light ray will take). Pose as a one variable optimization problem and solve analytically.

**Solution and Comments:**

You got the basic formulation if you arrived at Snell's law

\[
\frac{\sin \theta_p}{\sin \theta_q} = \frac{v}{w}
\]

It is challenging to find the solution in closed form. It is often best to get rid of unnecessary variables. In particular, by shifting the coordinate axis, one can get rid of \( x_p \), and one can reduce the problem to following form:

\[
v^2 - w^2 = \left(\frac{v y_q}{x - x_q}\right)^2 - \left(\frac{v y_p}{x}\right)^2
\]

which is of the form: \[
\frac{A}{(\alpha - 1)^2} - \frac{B}{\alpha^2} - C = 0 \] that Maple can handle.