1. Find the largest rectangle that can fit inside the ellipse:

\[
\left( \frac{x}{a} \right)^2 + \left( \frac{y}{b} \right)^2 = 1
\]

by posing a one variable optimization problem and solving for the minima analytically.

2. Consider the function \( f(x) = x^3 - x^2 + 2 \). Find and classify the stationary points using analytical methods.

3. Consider a beam of sectional modulus EI and length L that is pinned at both ends. A load P is applied at a distance L/3 from the left end and a load P/2 is applied at the center. Find the location and value of maximum deflection. (You need to review basic Mechanics of Materials)

4. (Diffraction Law of Optics). Let p and q be two points on the plane that lie on opposite sides of a horizontal axis. Assume that the speed of light from p to the horizontal axis is v, and from the horizontal axis to q is w. Find the fastest path from p to q (the path that a light ray will take). Pose as a one variable optimization problem and solve analytically.