1. Consider the function \( f(x, y) = x^2 - xy + 4y^2 + x - 3 \). Suppose a direction \( d = (1,1) \) is given. Find the corresponding conjugate direction. Using this pair show that you can find the minima from the origin in two steps.

2. Consider the minimization problem:
\[
\min f(x, y) = 0.5x^2 + 2.5y^2
\]
Starting the initial guess point \( (x, y) = (5,1) \) determine the next two points if one uses the conjugate gradient method.

3. Consider a quadratic function of arbitrary dimension \( f = \frac{1}{2} x^T Qx + x^T b + c \). Show that, one can start at any point \( x_0 \) and converge to the minima using the Newton's method in exactly one step.

4. Consider the pair of non-linear equations:
\[
\begin{align*}
x^2 + y^2 &= 1 \\
x^2 - 6x + y^2 &= -5
\end{align*}
\]
Starting the initial guess point \( x_0 = (2,2) \) determine the next two points if one uses the Newton's method to solve the above pair.

5. Consider the minimization problem:
\[
\min f(x) = 0.5 - xe^{-x^2}
\]
Starting the initial guess point \( x_0 = 1 \) determine the next two points if one uses the Newton's method to find the minima.