1. Consider the problem:
\[ P \begin{cases} \text{Min : } f(\bar{x}) \\ \text{s.t. } g(\bar{x}) \geq 0 \end{cases} \]
and the perturbed problem:
\[ P_\varepsilon \begin{cases} \text{Min : } f(\bar{x}) \\ \text{s.t. } g(\bar{x}) + \varepsilon \geq 0 \end{cases} \]
State and prove the relationship between the minimal functional values \( f^{\text{min}} \) and \( f^{\varepsilon \text{min}} \) for the two problems.

2. Prove that a quadratic function with positive definite Hessian is strictly convex.

3. Prove that the intersection of two convex regions is convex (use the definition of the convex region).

4. Consider the problem:
\[
\begin{align*}
\text{Min } J &= \int_0^1 \left( 3x^2 + 2x \left( \frac{dy}{dx} \right)^2 + 10xy \right) \, dx \\
y(0) &= y(1) = 0
\end{align*}
\]
State the weak and strong (Euler-Lagrange) optimality conditions.

5. Consider the problem:
\[
\begin{align*}
\text{Min } J &= \int_0^3 \left( 3 \left( \frac{dy}{dx} \right)^2 + 4x \right) \, dx \\
y(0) &= 0 \\
y(3) &= 2
\end{align*}
\]
Find the optimal solution.