Large-Scale Optimal Control of Interconnected
Natural Gas and Electrical Transmission Systems∗

Nai-Yuan Chiang and Victor M. Zavala†
Mathematics and Computer Science Division
Argonne National Laboratory, 9700 South Cass Avenue, Argonne, IL 60439, USA

Abstract

We present an optimal control model that captures spatiotemporal interactions between gas and electric transmission networks. We use the model to study flexibility and economic opportunities provided by coordination. A case study in the Illinois system reveals that coordination enables the delivery of significantly larger amounts of natural gas to the power grid. In particular, under a coordinated setting, gas-fired generators act as distributed demand response resources that can be controlled by the gas pipeline operator. This enables more efficient control of pressures and flows in space and time and overcomes delivery bottlenecks. We demonstrate that the additional flexibility not only benefits the gas operator but also leads to more efficient power grid operations and results in increased revenue for gas-fired power plants. We also use the optimal control model to analyze computational issues arising in these complex models. We demonstrate that the interconnected Illinois system gives rise to a highly nonlinear optimal control problem with 4,400 differential and algebraic equations and 1,040 controls that can be solved with a state-of-the-art sparse optimization solver.

Keywords: large scale; optimal control; natural gas; electricity; networks; dynamics

1 Introduction

Natural gas and power grid infrastructures are becoming increasingly interdependent. A major factor driving this situation is the increasing deployment of gas-fired power plants [1]. These plants are modular and less capital intensive compared with large, centralized generation facilities running on nuclear and coal fuel sources. In addition, gas-fired plants are more flexible and can quickly ramp up and down their power output. This flexibility becomes an asset as the share of intermittent solar and wind power increases. Moreover, the high availability of gas resulting from new fracking technologies has led to lower prices, making gas-fired plants economically more attractive.

An important feature of gas-fired generation (compared with other generation technologies) is that the fuel must be transported to power generation facilities in gaseous form through a sophisticated network of pipelines that span thousands of miles. A key advantage of this setting is that

∗Preprint ANL/MCS-P5348-0515.
†Corresponding Author: vzavala@mcs.anl.gov
significant amounts of gas can be stored inside the pipelines. The stored gas is distributed spatially along the pipelines and is normally referred to as line-pack [5]. Line-pack is used by pipeline operators to modulate variations of gas demands at multiple spatial points in intraday operations. Some of the strongest variations in gas demands are the result of on-demand start-up and shut-down of gas-fired power plants [16]. Modulating these variations is challenging because the fast release of line-pack at multiple simultaneous locations can trigger complex spatiotemporal dynamic responses that propagate hundreds to thousands of miles and that can take hours to stabilize. Therefore, line-pack management is performed by using sophisticated optimal control and pipeline simulation tools. These automation tools orchestrate the operation of a multitude of compressor stations distributed throughout the system with the objectives of satisfying demands, maintaining pressure levels, and minimizing compression costs [17].

An important issue faced by gas-fired power plants is that they compete for natural gas with industrial facilities and with local distribution companies (LDCs) that supply gas to urban areas. Therefore, natural gas cannot be guaranteed to be available at each power generation facility at all times. This limitation is particularly evident during the winter season when residential and office buildings require large amounts of gas for heating. An extreme manifestation of this issue was observed during the polar vortex of 2014 in which sustained low temperatures in the Midwest region of the U.S. led to high gas demands in urban areas and to equipment failures [10, 11]. These factors resulted in widespread shortages of natural gas in places as remote as California, Massachusetts, and Texas. These gas shortages in turn resulted in lost electrical generation capacity totaling 35 GW. At a value of lost load of 5,000 $/MWh, shortages of this magnitude represent economic losses of 175 million $/hr. The New England area alone lost 1.5 GW of power generation capacity [2]. The polar vortex also exposed market inefficiencies resulting from the increasing interaction between grid and gas systems. In particular, gas-fired plants required significant uplift payments from the independent system operators (ISOs). These payments compensated the power plants for the lost revenue resulting from the inability of the gas infrastructure to deliver fuel [6]. These operational and economic issues question the ability of the gas infrastructure to sustain additional gas-fired generation. This issue is important because, as we previously mentioned, gas-fired plants are essential to scale up renewable power generation.

We present an optimal control model to capture spatiotemporal interactions between gas and electric transmission systems. We use the model to investigate the economic and flexibility gains resulting from coordinating the dispatch of both systems. The resulting model is a large-scale and highly nonlinear optimal control model that we also use to assess the performance of state-of-the-art optimization tools and to identify sources of complexity. Our work extends previous work in the area in several ways. Optimal control of natural gas networks using high-resolution dynamic models has been reported by several researchers [19, 5, 21]. These studies do not consider coordination with power grid networks and treat power plants as exogenous demands. In this respect, we extend existing work by coupling high-fidelity dynamic gas models to power grid dispatch models. This allows us to gain new interesting insights; for instance, we demonstrate that significantly larger amounts of gas can be delivered to the power grid by controlling power plant gas demands. Researchers have also reported on optimization of interconnected power grid and natural gas networks but the dy-
namics of natural gas pipelines is neglected [9, 4, 3]. Steady-state models cannot capture line-pack storage dynamics and thus significantly underestimates the flexibility of the system in real-time operations. Consequently, steady-state models are more appropriate for long-term planning studies [18]. In this respect, our model seeks to better capture the flexibility provided by line-pack in real-time operations. Recent studies have also reported on co-optimization of gas and power grid transmission systems using dynamic gas models but these studies have been limited to very small and synthetic systems [12, 13]. In this respect, we extend existing work by focusing on real-sized networks and this enables us to obtain more accurate estimates of economic performance. In particular, we present a case study in the Illinois system that, to the best of our knowledge, is the largest study reported in the literature. Focusing on real-sized networks enables us to analyze computational issues preventing scalability of state-of-art optimization solvers to larger systems.

The paper is structured as follows. In Section 2 we present the power grid and gas side components of the model and their interconnection. In this section we also explain issues related to coordinated and uncoordinated dispatch settings. In Section 3 we present a case study using the Illinois system in which we compare the performance of coordinated and uncoordinated settings. We close the paper with concluding remarks and directions for future work.

2 Optimal Control Model

We consider a coupled dispatch model for electrical and natural gas transmission networks. We assume that coupling occurs only through the gas-fired plants that withdraw natural gas directly from the gas network to generate electricity for the power grid. Our model seeks to capture the effect of gas withdrawals on the gas network dynamics and seeks to determine how physical constraints can lead to shortages and decreased economic performance.

2.1 Power Grid Side

Economic dispatch is an optimal control problem that is solved by ISOs to balance supply and demand and to price electricity in intraday operations. We formulate the economic dispatch problem
as the following continuous-time optimal control problem:

\[
\begin{align*}
\min \ & \varphi_{\text{grid}} := \int_0^T \left( \sum_{i \in S} \alpha_i^s s_i(\tau) - \sum_{j \in D} \alpha_j^d d_j(\tau) \right) d\tau \quad (2.1a) \\
\text{s.t.} \ & \frac{ds_i(\tau)}{d\tau} = r_i(\tau) \quad (2.1b) \\
& \sum_{i \in S_n} s_i(\tau) - \sum_{j \in D_n} d_j(\tau) + \sum_{\ell \in L^{\text{rec}}_n} f_\ell(\tau) - \sum_{\ell \in L^{\text{snd}}_n} f_\ell(\tau) = 0, \ n \in N \quad (2.1c) \\
& f_\ell(\tau) = \beta_\ell \left( \theta_{\text{snd}(\ell)}(\tau) - \theta_{\text{rec}(\ell)}(\tau) \right), \ \ell \in L \\
& f^L_\ell \leq f_\ell(\tau) \leq \overline{f}_\ell, \ \ell \in L \\
& \theta_n \leq \theta_n(\tau) \leq \overline{\theta}_n, \ n \in N \\
& s_i \leq s_i(\tau) \leq \overline{s}_i, \ i \in S \\
& r_i \leq r_i(\tau) \leq \overline{r}_i, \ i \in S \\
& 0 \leq d_j(\tau) \leq d^{\text{target}}_j(\tau), \ i \in S \\
& d^{\text{gas,grid}}_i(\tau) = \eta_i \cdot s_i(\tau), \ i \in S_g. \quad (2.1j)
\end{align*}
\]

Here, \( \tau \in [0, T] \) denotes the time dimension and \( T \) is the final time. We define the sets of electricity suppliers (also referred to as power plants or generators) as \( S \), the set of electrical loads as \( D \), the set of network nodes as \( N \), and the set of transmission lines as \( L \). For each link \( \ell \in L \) we denote \( \text{snd}(\ell) \in N \) as the sending node and \( \text{rec}(\ell) \in N \) as the receiving node.

We define \( S_g \subseteq S \) as the subset of power plants that are fired by natural gas. The rest of the suppliers are either thermal or renewable power suppliers. We define \( S_n \subseteq S \) as the subset of suppliers connected to node \( n \) and \( D_n \subseteq D \) as the subset of loads connected to node \( n \). We define \( L^{\text{snd}}_n \subseteq L \) as the set of links originating from node \( n \) and \( L^{\text{rec}}_n \subseteq L \) as the set of lines ending in node \( n \).

The time profiles for power generation, delivered loads, flows, and voltage angles are denoted as \( s_i(\cdot), \ i \in S; d_j(\cdot), \ j \in D; f_\ell(\cdot), \ \ell \in L; \) and \( \theta_n, \ n \in N \), respectively. The time profiles for target loads are denoted as \( d^{\text{target}}_j(\cdot) \). The power flows are bounded by the capacity limits \( f^L_\ell, \overline{f}_\ell \), the angles are bounded by \( \theta_n, \overline{\theta}_n \), and the supply flows are bounded by \( s_i, \overline{s}_i \). The supply change rates are given by \( r_i(\cdot), \ i \in S \) and are bounded by the ramp limits \( \overline{r}_i, \overline{r}_i \).

The generation and demands costs are \( \alpha_i^s, \ i \in S \) and \( \alpha_j^d, \ j \in D \), respectively. In the case of inelastic demands, the demand costs are typically set to the value of lost load (VOLL). This value is typically in the range 1,000-10,000 $/MWh [7]. The dual variables of the network balance equation (2.1c) are the locational marginal prices, which we denote as \( \pi_n(\cdot), \ n \in N \).

We define \( d^{\text{gas,grid}}_i(\cdot), \ i \in S_g \) as the gas demands originating from the gas-fired plants and \( \eta_i, \ i \in S_g \) as the heat rates of the different power plants. The heat rate is a measure of the conversion efficiency of a power plant installation and is defined as the amount of fuel (in BTUs) needed to produce a KWh of electrical energy. The ideal heat rate is 3,412 BTU/KWh, which indicates a one-to-one conversion between fuel and electrical energy (efficiency of 100%). Consequently, the smaller the heat rate, the more efficient the technology. Different gas-fired plants can have different heat rates (combined cycle plants have much smaller heat rates than do simple-cycle plants). In fact, the
energy information administration (EIA) reports that the average heat rate for gas-fired plants has
been reduced steadily from 9,207 BTU/KWh in 2003 (efficiency of 37%) to 7,948 BTU/KWh in 2013
(efficiency of 42.9%) because of the adoption of new technologies. In comparison, the average heat
rate for coal power plants was reported to be 10,459 BTU/KWh (efficiency of 32%) in 2013 and this has
remained at similar levels since 2003 (see http://www.eia.gov/electricity/annual/html/
epa_08_01.html).

2.2 Natural Gas Side

Line-pack management is an optimal control problem that is solved by gas pipeline operators to
balance supply and demand while minimizing compression costs and maintaining pressure levels
throughout the network [17, 5, 19]. We refer to this problem as the gas dispatch problem. We for-
mulate this problem as the optimal control problem constrained by partial differential and algebraic
equations (PDAEs) presented in equation (2.2). We define the sets describing the topology of the
gas network using a notation similar to that of the power grid network but we add the superscript
gas to differentiate them. For instance, $L^{gas}$ is the set of gas links (pipelines). We define the set of
passive and active pipeline segments as $L_p^{gas} \subseteq L^{gas}$ and $L_a^{gas} \subseteq L^{gas}$, respectively. Active links
have compression stations at the node of origin $snd(\ell)$ whereas passive links are only transport links.

Symbol $x$ denotes the axial dimension and the spatial domain of each pipeline segment is denoted as
$X_\ell := [0, L_\ell]$, $\ell \in L^{gas}$. The length, diameter, and transversal area of the pipelines are denoted as $L_\ell$, $D_\ell$, and $A_\ell$, respectively.

The nonlinear transport equations (2.2b)-(2.2c) capture the spatiotemporal dynamics of flow and
pressure, which are necessary to describe the dynamics of the line-pack storage. Steady-state mod-
els cannot capture this behavior. Capturing line-pack dynamics in intraday operations is important
because gas transport is slow (on the order of hours) [14]. The boundary conditions for the transport
equations are given by (2.2e)-(2.2f). The balance at each node is given by (2.2h).

The spatiotemporal profiles of flows, pressures, and densities in segment $\ell$ are denoted as $f_\ell^{gas}(\cdot, \cdot)$, $p_\ell^{gas}(\cdot, \cdot)$, and $\rho_\ell^{gas}(\cdot, \cdot)$, respectively. We note that the gas pressure and density are related through the gas speed
of sound $c$. Consequently, the transport equations can be written in terms of flow and pressure alone
by substituting the constraint (2.2d) into (2.2b)-(2.2c). We write the model in terms of flow, pressure,
and density to enhance readability. The node pressures are given by $\theta_n(\cdot)$, $n \in N^{gas}$. Symbols
$\Delta \theta_\ell^{gas}(\cdot)$, $\ell \in L_a$ denote the compressor pressure increments of the active links. Accordingly, $\theta_{snd(\ell)}(\cdot)$ are the inlet pressures for the compressors and $\theta_{snd(\ell)}(\cdot) + \Delta \theta_\ell^{gas}(\cdot)$ are the outlet pressures. The com-
pression power in the active links is denoted as $P_\ell^{gas}(\cdot)$ and the costs of compression are $\alpha_{P,gas}^{P,gas}$. The
compression power (assuming ideal gas) is computed from (2.2i).

The gas supply flows are denoted as $s_\ell^{gas}(\cdot)$, the delivered gas demands are $d_\ell^{gas}(\cdot)$, the gas
demand targets are $d_{\ell^{gas,target}}(\cdot)$, and the actual delivered gas demands are denoted as $d_\ell^{gas}(\cdot)$. Symbol
$\alpha_{d,gas}$ denotes the value of the delivered gas and $\alpha_{P,gas}^{P,gas}$ is the cost of compression. From the structure
of the objective function (2.2a) we thus see that the operator seeks to maximize the amount of gas
delivered at the multiple demand locations while minimizing the total compression cost. We assume
the supply flows to be fixed and, consequently, they do not appear in the objective.
\[
\min \varphi^{\text{gas}} := \int_0^T \left( \sum_{\ell \in \mathcal{L}_a} \alpha^\ell \varphi_{\text{gas}}^\ell(\tau) - \sum_{j \in \mathcal{D}^{\text{gas}}} \alpha_j^{\ell,\text{gas}} \varphi_j^{\text{gas}}(\tau) \right) d\tau 
\]  
(2.2a)

s.t.

\[
\frac{\partial p_{\text{gas}}(x, \tau)}{\partial \tau} + \frac{1}{A^\ell} \frac{\partial f_{\text{gas}}^\ell(x, \tau)}{\partial x} = 0, \quad \ell \in \mathcal{L}^{\text{gas}}, \; x \in \mathcal{X}^{\ell} 
\]  
(2.2b)

\[
\frac{1}{A^\ell} \left( \frac{\partial f_{\text{gas}}^\ell(x, \tau)}{\partial \tau} + \frac{\partial f_{\text{gas}}^\ell(x, \tau)}{\partial x} + \frac{8\lambda^\ell}{\pi^2 D_x^\ell} \frac{f_{\text{gas}}^\ell(x, \tau)}{p_{\text{gas}}^\ell(x, \tau)} \right) = 0, \quad \ell \in \mathcal{L}^{\text{gas}}, \; x \in \mathcal{X}^{\ell} 
\]  
(2.2c)

\[
p_{\text{gas}}^\ell(0, \tau) = \varphi_{\text{gas}}_{\text{rec}}(\tau), \quad \ell \in \mathcal{L}^{\text{gas}} 
\]  
(2.2d)

\[
p_{\text{gas}}^\ell(0, \tau) = \varphi_{\text{gas}}_{\text{snd}}(\tau), \quad \ell \in \mathcal{L}^{\text{gas}} 
\]  
(2.2e)

\[
p_{\text{gas}}^\ell(0, \tau) = \varphi_{\text{gas}}_{\text{rec}}(\tau) + \Delta\varphi_{\text{gas}}^\ell(\tau), \quad \ell \in \mathcal{L}^{\text{gas}} 
\]  
(2.2f)

\[
\sum_{\ell \in \mathcal{L}^{\text{gas},\text{rec}}} f_{\text{gas}}^\ell(\mathcal{L}^{\ell}, \tau) - \sum_{\ell \in \mathcal{L}^{\text{gas},\text{snd}}} f_{\text{gas}}^\ell(0, \tau) + \sum_{i \in \mathcal{S}^{\text{gas}}} \varphi_{i}^{\text{gas}}(\tau) - \sum_{j \in \mathcal{D}^{\text{gas}}} \varphi_{j}^{\text{gas}}(\tau) = 0, \quad n \in \mathcal{N}^{\text{gas}} 
\]  
(2.2g)

\[
F_{\text{gas}}^\ell(\tau) = \varphi_{\text{gas}} T^{\text{gas}} \cdot f_{\text{gas}}^\ell(0, \tau) \left( \left( \frac{\varphi_{\text{gas}}_{\text{rec}}(\tau) + \Delta\varphi_{\text{gas}}^\ell(\tau)}{\varphi_{\text{gas}}_{\text{snd}}(\tau)} \right)^{\frac{n-1}{n}} - 1 \right), \quad \ell \in \mathcal{L}^{\text{gas}} 
\]  
(2.2h)

\[
\int_0^{\mathcal{L}^{\ell}} f_{\text{gas}}^\ell(x, T) dx \geq \int_0^{\mathcal{L}^{\ell}} f_{\text{gas}}^\ell(x, 0) dx, \quad \ell \in \mathcal{L}^{\text{gas}} 
\]  
(2.2i)

\[
\varphi_{n}^{\text{gas}} \leq \varphi_{n}^{\text{gas}}(\tau) \leq \varphi_{n}^{\text{gas}}, \quad n \in \mathcal{N} 
\]  
(2.2j)

The line-pack stored in each segment \( \ell \in \mathcal{L} \) at time \( \tau \) is given by

\[
\int_0^{\mathcal{L}^{\ell}} f_{\text{gas}}^\ell(x, \tau) dx. 
\]  
(2.3)

We thus have that (2.2) is a periodicity constraint that requires the line-pack stored in each segment at the final time \( \tau = T \) to be at least as large as that at the initial time \( \tau = 0 \). In the absence of this constraint, the system will tend to deplete the line-pack in order to minimize compression power. This depletion is undesirable because line-pack is necessary for the next cycle of operation (i.e., the stored gas has economic value) [17].

Constraints (2.2k) are used to model suction and discharge pressures at the compressor stations as well as minimum and maximum pressure levels at demand points. We denote the compression component of the total gas system cost \( \varphi^{\text{gas}} \) as

\[
\varphi^{\text{gas,comp}} := \int_0^T \sum_{\ell \in \mathcal{L}_a} \alpha^\ell \varphi_{\text{gas}}^\ell(\tau) d\tau 
\]  
(2.4)
2.3 Coordinated and Uncoordinated Settings

The coupling between infrastructures is given by the following constraints:

\[
\begin{align*}
\beta_j^\text{gas,target}(\tau) &= \beta_{\text{grid}}^\text{gas,grid}(\tau) + \beta_j^\text{gas,base}(\tau), \; j \in \mathcal{D}^\text{gas}, \; \tau \in [0, T] \quad (2.5a) \\
\beta_{\text{grid}}^\text{gas,grid}(\tau) &\leq \beta_j^\text{gas}(\tau) - \beta_j^\text{gas,base}(\tau), \; j \in \mathcal{D}^\text{gas}, \; \tau \in [0, T]. \quad (2.5b)
\end{align*}
\]

The first constraint states that the gas demand targets for the gas infrastructure are given by the gas demands of the gas-fired power plants plus an exogenous base gas demand that arises from industrial facilities and/or LDCs serving urban areas. Here, \(\beta_{\text{grid}}^\text{gas}(\tau)\) denotes the power plant corresponding to the gas demand \(j \in \mathcal{D}^\text{gas}\). The second constraint states that the gas used by the power plants cannot physically exceed the gas delivered by the gas infrastructure. We use this second constraint to model situations in which the gas infrastructure is physically constrained and thus the delivered demand \(\beta_j^\text{gas}(\cdot)\) cannot match the target demand \(\beta_j^\text{gas,target}(\cdot)\). The solution of the grid and gas models together with the coupling constraints (2.5) gives the coordinated dispatch model.

To establish a basis for comparison, we consider an uncoordinated setting in which the two infrastructures do not dispatch jointly. This is simulated by first solving the economic dispatch problem for the power grid (2.1) to set the predicted generation \(s_i(\cdot), \; i \in \mathcal{S}\), the natural gas demand targets \(\beta_{\text{grid}}^\text{gas,grid}(\cdot), \; j \in \mathcal{D}_g\), and the predicted locational marginal prices \(\pi_n, \; n \in \mathcal{N}\). Having the gas demand targets, we solve the gas dispatch problem (2.2) to maximize the gas delivered and minimize compression costs. The solution of this problem sets the realized gas demands delivered to the gas-fired power plants \(\beta_j^\text{gas}(\cdot) - \beta_j^\text{gas,base}(\cdot)\). Because the realized gas demands might not be able to match the power grid targets, we solve the economic dispatch problem for the power grid (2.1) again to determine the realized generation schedule and locational marginal prices corresponding given the realized delivered gas demands. We denote the realized power generation schedules as \(s_i^\text{real}(\cdot)\) and the prices as \(\pi_n^\text{real}(\cdot)\). Differences between the target and delivered gas demands will introduce a difference between the predicted and realized generation schedules and prices. When the gas-fired power plants cannot obtain the total gas requested, they will need to curtail power and must pay for the unserved electricity generation at the realized price \([15]\). In such a case, the revenue for the power plants is given by

\[
\mathcal{R}_i := \int_0^T \left( \pi_{\text{stn}(i)}(\tau) s_i(\tau) + s_i^\text{real}(\tau) (\pi_n^\text{real}(\tau) - \pi_{\text{stn}(i)}(\tau)) - \alpha_i s_i^\text{real}(\tau) \right) d\tau, \quad i \in \mathcal{S}_g. \quad (2.6)
\]

Here, \(\text{stn}(i) \in \mathcal{N}\) denotes the node at which supplier \(i \in \mathcal{S}_g\) is connected to. The total revenue for the gas-fired generators is denoted by \(\mathcal{R} = \sum_{i \in \mathcal{S}_g} \mathcal{R}_i\).

When the requested and delivered gas demands coincide we have that the predicted and realized generation schedules coincide (i.e., the predicted generation schedule is feasible to the gas system). Consequently, we have that \(\pi_{\text{stn}(i)}^\text{real} = \pi_{\text{stn}(i)}(\tau)\) and the revenue reduces to

\[
\mathcal{R}_i = \int_0^T \left( \pi_{\text{stn}(i)}(\tau) s_i(\tau) - \alpha_i s_i^\text{real}(\tau) \right) d\tau, \quad i \in \mathcal{S}_g. \quad (2.7)
\]

Note that, in the absence of uncertainty, this case also corresponds to the coordinated setting.
Figure 1: Illinois electrical (blue) and gas (red) transmission systems. Black dots are locations of gas-fired power plants.

3 Illinois Case Study

We use a case study in Illinois to illustrate some of the insights that can be gained with the proposed model. The Illinois power grid transmission system is built on a realistic data set that we have used for previous studies [23]. The system comprises 2,522 lines, 1,908 nodes, 870 demands points, and 225 generators points (153 gas-fired generators). Because of the difficulty in obtaining natural gas infrastructure data, we construct a simulated natural gas network system using the basic topology reported by the EIA (see http://www.eia.gov/pub/oil_gas/natural_gas/analysis_publications/ngpipeline/midwest.html) and by using engineering insight to ensure gas supply to all the gas-fired power plants under nominal conditions. The gas network designed comprises 215 pipeline segments, 157 nodes, 12 compression stations, and 4 supply points. The resulting network in sketched in Figure 1.

3.1 Economic Issues

We compare economic performance for the infrastructures under coordinated and uncoordinated settings. We use a time horizon of 24 hours. The results are summarized in Table 1. In our simulations, the electrical loads were always satisfied; consequently, we report only the generation cost component of the grid cost $\phi_{grid}$. From the results we make the following observations:

- Under a coordinated setting the power cost decreases by 0.38% which represents a total of $140,000. The gas cost decreases by 7%, which corresponds to a total of $970,000.
Under an uncoordinated setting only 96% of the gas requested is delivered. At a gas price of 3 $/MMBTU (see http://www.eia.gov/naturalgas/weekly/), the total undelivered gas has an economic value of $599,000.

Under a coordinated setting the compression cost increases by 17.4%. This is the result of an increased amount of gas delivered to the power plants. In particular, 7% more gas is delivered under the coordinated setting. At a gas price of 3 $/MMBTU, the value of the additional gas delivered is $1,070,000. Note that the total increase in compression cost is negligible compared to the additional value of the delivered demand.

Under a coordinated setting the revenue for the gas-fired generators increases by 27%, which corresponds to a total of $800,000. This the result of the additional gas delivered and the decreased revenue penalties resulting from coordination.

Table 1: Economic performance under coordinated and uncoordinated settings (scm= standard cubic meters and M$=million U.S. dollars).

<table>
<thead>
<tr>
<th></th>
<th>$\phi_{grid}$ [M$]$</th>
<th>$\phi_{gas}$ [M$]$</th>
<th>$\phi_{gas,comp}$ [$]$</th>
<th>$d^{gas,target}$ [scm $\times 10^{-6}$]</th>
<th>$d^{gas}$ [scm $\times 10^{-6}$]</th>
<th>$R$ [M$]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncoord</td>
<td>36.54</td>
<td>-13.52</td>
<td>28,618</td>
<td>141.25</td>
<td>135.54</td>
<td>2.70</td>
</tr>
<tr>
<td>Coord</td>
<td>36.40</td>
<td>-14.54</td>
<td>33,600</td>
<td>145.74</td>
<td>145.74</td>
<td>3.50</td>
</tr>
</tbody>
</table>

Figure 2: Requested (blue solid line) and realized (green dotted line) gas demands for 16 power plants under uncoordinated setting.
Figure 3: Requested (blue solid line) and realized (green dotted line) gas demands for 16 power plants under coordinated setting.

Figure 4: Power compression profiles under uncoordinated (blue) and coordinated (green) settings for 16 different compressor stations.
We can thus see that both the gas and power grid sides benefit from coordination (i.e., *the objectives of the gas and grid operators do not compete*). Moreover, gas-fired generators increase their revenue. We can explain the decreased performance under an uncoordinated setting from the fact that the power grid operator cannot easily determine how much gas can the gas network deliver at different spatial locations and at different times. Thus, the power grid operator can be overly optimistic (as in the case presented) or pessimistic about the amount of gas that can actually be delivered. This situation is clearly illustrated in Figure 2, where we present the target and realized gas demands for 16 different gas-fired generators for the uncoordinated setting. Note that the gas network cannot deliver the total amount of gas requested at four locations. The resulting error in the prediction introduces a penalty for both the power grid and the gas-fired generators. In particular, the power grid operator has to dispatch more expensive power plants, resulting in higher a generation cost and the power plants have to pay for the unserved generation. Note also that, even if the gas operator knows the gas demands of the power grid in advance, it cannot guarantee to satisfy such demands due to physical constraints. In other words, the spatiotemporal gas demands policies emanating from the power grid dispatch plan can be infeasible to the gas infrastructure.

From Figure 2 we can also see that several gas-fired generators are dispatched aggressively under an uncoordinated setting (i.e., some gas demands are step functions). This is evident from panels 1, 14, 15, and 16 (the panels are numbered row-wise starting from the upper left corner). Line-pack storage in this case is sufficient to track the steep changes in gas demand. Doing so, however, comes at the expense of decreased flexibility at other locations and this leads to unserved demands. From panels 1, 14, 15, and 16 of Figure 3 we also see that dispatch under a coordinated setting is smoother (i.e., demands are ramps) and this significantly enhances the flexibility to deliver gas at other locations. We observe that, under a coordinated setting, gas-fired generators act as distributed demand response resources that the gas operator can use to better control network pressures and flows and thus avoid delivery bottlenecks. In other words, *gas-fired power plants become assets rather than liabilities*. As a result, all the demand targets can be met and significantly more gas can be delivered compared to
the uncoordinated setting. This clearly illustrates the increased physical flexibility gained by coordination.

Figure 4 presents the compression power profiles under coordinated and uncoordinated settings for 16 different compressor stations. We observe that more compressors operate at full capacity under a coordinated setting, a situation that is desirable from an efficiency standpoint (i.e., it is inefficient to operate compressors at partial loads and to ramp them up and down continuously from an equipment lifetime and emissions standpoint). From Figure 5 we see that more compression power is used under the coordinated setting but the compression profile is much flatter.

Figures 6 and 7 present axial flow profiles at 16 different pipeline segment for the uncoordinated and coordinated settings, respectively. Each panel presents the time profiles at 10 different axial positions for a given pipeline segment. We can see how the different dispatch plans drastically change the flow profiles of the network. This result indicates that power grid dispatch decisions can significantly influence the spatiotemporal dynamics of the entire gas infrastructure.

![Figure 6: Spatiotemporal flow profiles for 16 different pipeline segments for uncoordinated setting.](image)

3.2 Computational Issues

We now discuss issues related to the implementation and solution of the optimal control model.

3.2.1 Model Implementation

We discretize the continuous-time grid model (2.1) using an implicit Euler discretization scheme. We discretize the gas model (2.2) in time using an implicit Euler scheme and in space using a forward finite-difference scheme. The discretized power grid problem is a linear program, whereas the discretized gas problem is a nonlinear programming problem (NLP). The coupled problem is an NLP.
We implement the coupled and decoupled models independently using the algebraic modeling language AMPL. The use of an algebraic modeling language is key because it enables us to obtain exact first and second-order derivative information for the gas model. Exact derivatives are essential for efficient handling problems with many degrees of freedom as those arising in interconnected models [22].

We couple the infrastructure models by using AMPL’s suffix capability. This capability is useful for conveying structure of special problem classes to optimization solvers [24]. In our context, this capability enables us to create the coupled gas-electric model while avoiding sharing information (e.g., network topology data) between modelers. This is beneficial from a modeling standpoint because it allows us to build, debug, and test each model component independently. Using this capability we can also use legacy power grid and gas dispatch models (with different formulations or different network domains) developed by other users. The coupled gas-electric problem expressed in this form has the structure

\[
\begin{align*}
\min & \quad f_{grid}(w_{grid}) + f_{gas}(w_{gas}) \\
\text{s.t.} & \quad c_{grid}(w_{grid}) \geq 0 \quad (3.8a) \\
& \quad c_{gas}(w_{gas}) \geq 0 \quad (3.8b) \\
& \quad \Pi_{gas} w_{gas} + \Pi_{grid} w_{grid} \geq 0. \quad (3.8c)
\end{align*}
\]

Here, \( w_{grid} \) are all the variables in the grid model after discretization, and \( w_{gas} \) are all the variables in the gas model. This problem becomes decomposable if the coupling constraints \((3.8d)\) are removed. The coupling constraints correspond to \((2.5)\) and we note that all that is needed to form the coupling constraints are the matrices \( \Pi_{gas} \) and \( \Pi_{grid} \). These are trivial matrices (containing only zeros and ones) that can be constructed by using AMPL suffixes.
3.2.2 Model Solution

We solved the coupled and uncoupled models using a 24-hour time horizon with 1-hour time resolution. In our base implementation we discretize each pipeline segment using \(N_x = 10\) finite-difference points. This gives rise to an optimal control problem with 2,400 differential equations, 2000 algebraic equations, and 1040 controls. After full discretization, the coordinated problem is an NLP with 249,919 variables, 224,292 equality constraints, and 154,093 inequality constraints. The problem has a total of 25,627 degrees of freedom.

We solve the NLP using the interior-point solver PIPS-NLP [8], which uses the filter line-search algorithm implemented in IPOPT [20]. PIPS-NLP is used to experiment with different linear algebra solvers and modeling interfaces. In all our experiments we use MA57 to factorize the Karush-Kuhn-Tucker (KKT) matrix. We use the nested dissection ordering implemented in the METIS package, which has been shown to be efficient at solving KKT systems arising from optimization problems with PDAE constraints [22]. All our experiments are performed on a single 2.7 GHz Intel Core i7 processor.

The computational results are summarized in Table 2. The solution time for the base problem is approximately 40 minutes. While the results are acceptable for off-line analysis, such times are too long to consider an actual real-time implementation of the model (which would probably need to be solved every 5 minutes to be compatible with current economic dispatch practices). The long solution times are partially attributed to the high nonlinearity of the gas network model which induces negative curvature. In the presence of negative curvature, the Hessian of the Lagrangian needs to be regularized (convexified) several times during the search and each regularization attempt requires an additional factorization. In particular, in Table 2 we can see that the total number of iterations is 232 while the total number of factorizations is 311, which indicates that 79 regularization attempts are needed.

We also attribute the long solution times to the complexity of the linear algebra system. In particular, we noticed that MA57 introduced a significant amount of fill-in, an indication of tight connectivity of the algebraic equations. We attribute this to the complexity induced by the PDAEs coupled with the network equations. To confirm this observation, we performed an additional experiment in which we perturbed the gas network topology by eliminating a single pipeline. This action splits the Illinois gas network into two independent subnetworks (the power grid topology remains intact). The results for the base and the perturbed topologies are presented in Table 2. The time per iteration is decreased by 18% under the perturbed topology. We also note a significant reduction in the number of iterations and regularizations which indicates that the nonlinearity is ameliorated with a change in topology. In particular, for the base topology we require 311/232=1.34 regularizations per iteration, whereas for the perturbed topology we require 153/136=1.125.

We next analyze the effects of the discretization resolution on the model results. We compare the results using our base implementation with \(N_x = 10\) spatial points per pipeline and a low resolution implementation with \(N_x = 3\) spatial points per pipeline. This low-resolution problem is an optimal control problem with 2800 DAEs and 1040 controls. After full discretization, this gives an NLP with 141,559 variables, 115,932 equality constraints, and 157,765 inequality constraints. The problem has a
Table 2: Computational results for coupled problems for base and perturbed topologies.

<table>
<thead>
<tr>
<th></th>
<th>Iter.</th>
<th>Solution Time [sec]</th>
<th>Factorizations [-]</th>
<th>Time/Factorization [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base Topology</strong> ($N_x = 10$)</td>
<td>232</td>
<td>2401.01</td>
<td>311</td>
<td>7.72</td>
</tr>
<tr>
<td><strong>Perturbed Topology</strong> ($N_x = 10$)</td>
<td>136</td>
<td>968.47</td>
<td>153</td>
<td>6.32</td>
</tr>
<tr>
<td><strong>Base Topology</strong> ($N_x = 3$)</td>
<td>157</td>
<td>573.68</td>
<td>188</td>
<td>3.05</td>
</tr>
</tbody>
</table>

We also compare the economic performance of the low- and high-resolution models for the coordinated and uncoordinated settings. The results are presented in Table 3. The amount of compression power in the uncoordinated case is underestimated by 21%, but the impacts on the rest of the metrics are not significant. This indicates that a low discretization resolutions can adequately capture the overall system behavior and can be used to obtain approximate solutions. Despite these improvements, however, we can see that state-of-the-art tools hit their limits of performance on small regional-scale networks and will not be capable of handling ISO-sized domains. Based on the estimates obtained for the Illinois system, we anticipate such NLPs to have tens of millions of variables and constraints. Specialized techniques based on decomposition and adaptive discretizations are needed to address models of this magnitude.

Table 3: Economic performance under low- and high-resolution discretizations.

<table>
<thead>
<tr>
<th></th>
<th>$\varphi_{grid}$ [M$]</th>
<th>$\varphi_{gas}$ [M$]$</th>
<th>$\varphi_{gas,comp}$ [$]$</th>
<th>$d_{gas,target}$ [scm $\times 10^{-6}$]</th>
<th>$d_{gas}$ [scm $\times 10^{-6}$]</th>
<th>$R_{gas}$ [M$]$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Uncoord</strong> ($N_x = 10$)</td>
<td>36.54</td>
<td>-13.52</td>
<td>28,618</td>
<td>141.25</td>
<td>135.54</td>
<td>2.70</td>
</tr>
<tr>
<td><strong>Uncoord</strong> ($N_x = 3$)</td>
<td>36.51</td>
<td>-13.57</td>
<td>23,592</td>
<td>141.12</td>
<td>135.96</td>
<td>2.74</td>
</tr>
<tr>
<td><strong>Coord</strong> ($N_x = 10$)</td>
<td>36.40</td>
<td>-14.54</td>
<td>33,600</td>
<td>145.74</td>
<td>145.74</td>
<td>3.50</td>
</tr>
<tr>
<td><strong>Coord</strong> ($N_x = 3$)</td>
<td>36.39</td>
<td>-14.55</td>
<td>33,356</td>
<td>145.83</td>
<td>145.83</td>
<td>3.48</td>
</tr>
</tbody>
</table>

4 Conclusions

We have presented an optimal control model for integrated gas-electric infrastructures. We used the model to demonstrate that significant improvements in economic performance and flexibility can be gained by coordinated dispatch. Using a large-scale study we demonstrated that, under a coordinated setting, it is possible to deliver significantly larger amounts of gas to the power grid.
and to improve the revenue of gas-fired plants. We observe that, under a coordinated setting, power plants act as controllable demand response resources that can be used by the gas pipeline operator to better control pressure and flows in space and time. This allows the gas operator to bypass delivery bottlenecks. We also used our model to illustrate that power dispatch policies can strongly influence the flow dynamics of the gas infrastructure. In addition, we found that the state-of-the-art tools can adequately address regional-scale networks but are insufficient to address national-scale networks. As part of future work, we will develop scalable linear algebra strategies based on decomposition and multi-grid techniques to address such problems. We will also develop models that capture transient stability of the power grid and that capture other dependencies between infrastructures. For instance, we will model dual-drive compressors that can run on both natural gas and electricity.

Acknowledgments

This material is based upon work supported by the U.S. Department of Energy, Office of Science, under Contract No. DE-AC02-06CH11357.

References


The submitted manuscript has been created by the University of Chicago as Operator of Argonne National Laboratory (“Argonne”) under Contract No. DE-AC02-06CH11357 with the U.S. Department of Energy. The U.S. Government retains for itself, and others acting on its behalf, a paid-up, nonexclusive, irrevocable worldwide license in said article to reproduce, prepare derivative works, distribute copies to the public, and perform publicly and display publicly, by or on behalf of the Government.