IIE Transactions (2009) **41**, 893–904
Copyright © "IIE"
ISSN: 0740-817X print / 1545-8830 online
DOI: 10.1080/07408170902906027

Surrogate modeling of dimensional variation propagation in multistage assembly processes

JEAN-PHILIPPE LOOSE[1], NAN CHEN[2] and SHIYU ZHOU[2,*]

[1]*Cisco Systems, Inc., 170 West Tasman Drive, San Jose, CA 95134, USA*
[2]*Department of Industrial and Systems Engineering, The University of Wisconsin – Madison, Madison, WI 53706, USA*
E-mail: szhou@engr.wisc.edu

Received March 2008 and accepted January 2009

In assembly process control and design optimization, it is critical to establish a mathematical model that describes the relationship between the dimensional quality of the final product and the various process parameters (e.g., the fixture layout and locator position deviation). This article presents a surrogate modeling methodology for multistage assembly processes to characterize the relationship between fixture layout and product dimensional quality. The mathematical structure of the model is derived from a physical analysis based on first principles and then the parameters of the model are identified using data from computer experiments. The resulting surrogate model can enable fixture layout optimization in process planning. A comprehensive case study of a multistage assembly process is also presented to demonstrate the effectiveness and high fidelity of the developed method.

Keywords: Surrogate modeling, variation propagation, multistage assembly

Notation

$\mathbf{t}_i$ = coordinates of the ith locator in a reference coordinate system;
$\mathbf{v}_i$ = small deviations of the ith locator from its nominal positions;
$\mathbf{U}$ = the small tolerance on the variation of the locator position;
$\mathbf{q}$ = location and orientation deviations of the parts;
$\mathbf{q}_0$ = nominal location and orientation of the part;
$\mathbf{s}_i$ = process design parameters;
$\boldsymbol{\gamma}$ = parameters of the surrogate model for a single part;
$\mathbf{n}_i$ = outgoing norm vector at the ith locator nominal position;
$\boldsymbol{\beta}$ = parameters of the a single rational function;
$\hat{\boldsymbol{\beta}}$ = least square estimates of parameter $\boldsymbol{\beta}$;
$\mathbf{J}$ = Jacobian matrix of the constraints of the locators;
$\mathbf{A}$ = coordinate transformation matrix from global coordinate to reference coordinate;
$\mathbf{H}_q$ = homogeneous transformation matrix of reference system from deviated position to nominal position;
$(\mathbf{A})_{i,j}$ = the element at the ith row and jth column of matrix $\mathbf{A}$;
$N^k(\mathbf{t})$ = kth-order polynomial function of variable $\mathbf{t}$;
q_i = the ith element of $\mathbf{q}$;
$\mathbf{a}_i^j$ = the jth rational components of q_i;
$\mathbf{dP}_i$ = dimensional deviation of mating point i propagated from previous stage.

*Corresponding author

1. Introduction

In recent years, dimensional variation reduction has become a crucial engineering objective during all stages of a product life cycle to maintain high-quality products while achieving ever shorter time-to-market requirements. Consequently, the modeling of dimensional variation propagation in multistage manufacturing processes has drawn significant attention. The dimensional variation propagation models are mathematical descriptions of the relationship between the dimensional quality of the final product and the various process parameters (e.g., the fixture layout, locator position deviation and the inaccuracy in machine geometry) and provide a quantitative basis for process design optimization and process monitoring and diagnosis.

The existing approaches for variation modeling in multistage manufacturing processes can be classified into two categories: analytical approaches and numerical approaches. In the analytical approaches, the modeling of the variation propagation is based on a physical analysis of the basic underlying operations of complicated manufacturing processes. A set of closed-form equations describing the relationship between the process variation sources (e.g., the variation of the positions of fixture locators and the variation of the incoming raw parts) and the product dimensional quality for each manufacturing stage is first derived; then, these equations are linked together to form the overall model for the complete multistage process. To make the derivation analytically tractable, the higher-order terms are often ignored in the derivation through a linearization procedure. Analytical variation propagation models have been successfully derived for assembly processes (Shiu *et al.*, 1996; Jin and Shi, 1999; Ding *et al.*, 2000; Ceglarek *et al.*, 2004) and machining processes (Huang *et al.*, 2000; Djurdjanovic and Ni, 2001; Zhong *et al.*, 2002; Zhou *et al.*, 2003; Loose *et al.*, 2007). These analytical models provide a theoretical foundation for process monitoring and diagnosis to identify the major variation sources in the process (Ceglarek and Shi, 1996, 1999; Ding *et al.*, 2002a; Ding, Zhou and Chen, 2005; Li and Zhou, 2006; Li *et al.*, 2007), process-oriented tolerance allocation (Ding *et al.*, 2002b; Ding, Jin, Ceglarek and Shi, 2005) and sensor distribution optimization (Khan and Ceglarek, 2000; Ding *et al.*, 2003). In the numerical approaches, practitioners rely on computer simulation to describe the underlying physical relationships between the product quality and process parameters. Several software packages, such as Tecnomatix, Sigmetric and Dimensional Control Systems (DCS), are available to simulate the variation propagation in multistage assembly processes, particularly in automotive assembly processes. These simulation models can often describe very complicated interactions within the manufacturing process and provide realistic results for large-scale systems. However, given the very large number of parameters typically included in an analysis, it is often very time-consuming to establish a simulation model and for certain physical processes, it is also very time-consuming to finish one simulation run. Thus, these models are currently used most in process design validation.

From a brief review of the existing modeling approaches for variation propagation, it can be seen that a common shortcoming of existing approaches is that they only describe the variation propagation under fixed process design settings, such as fixture layout. In other words, many very important process design parameters (e.g., the nominal positions of the fixture locators) are treated as constant values in these models. Thus, these models cannot be used in a process design optimization on these parameters. Although, in theory, all design parameters can be treated as free variables instead of known constant values in the analytical approaches, the extremely large number of design parameters and their complex interactions in multistage manufacturing processes make it impractical to analytically derive useful closed-form equations describing the variation propagation and include all design parameters as free variables. Similarly, it is impractical to use the simulation models in process design optimization because it is often time-consuming to change the process design settings in the simulation and also time-consuming to run the simulation. Consequently, experience-based trial-and-error methods are still commonly used in practice for design optimization of manufacturing processes. Clearly, these methods are costly and error-prone. There is an urgent need to develop a modeling technique for dimensional variation propagation that can take the large number of process design parameters into consideration.

The surrogate modeling technique which is based on computer simulation has recently become a popular method in engineering design. The basic idea of surrogate modeling is to first run a set of controlled computer simulation experiments. In the second step, based on the simulation results, a statistical model is established to describe the relation between inputs and outputs. Different types of models have been proposed as surrogate models (also referred to as metamodels in the literature). The most common one is based on polynomial functions to represent linear response surfaces (Myers and Montgomery, 1995). Its main limitation is that only a small number of parameters can be typically included in the analyses. Sacks *et al.* (1989) proposed the use of a stochastic model borrowed from spatial statistics called a Kriging model. In this model, the deterministic response from a simulated experiment is determined as the sum of a regression function, acting as global approximant to the data and a random process acting as local perturbation to interpolate the data. Kriging models have been successfully applied to fields as diverse as aerospace engineering design (Simpson *et al.*, 1998) and electrical engineering (Sacks *et al.*, 1989). Additional models, such as the radial basis functions, multivariate adaptive regression splines and also Neural Networks (NNs) have been employed successfully as metamodels. For a complete review of these models, the authors refer the readers to a set of review papers (Jin *et al.*, 2001; Simpson *et al.*, 2001; Chen *et al.*, 2006; Wang and Shan, 2007). The aforementioned models can then be used within an optimization routine in place of the complicated simulation model to find the best combination of input parameters within a prespecified design space leading to an acceptable set of output parameters. The obtained results are then validated using the simulation package and the decision is made to execute further runs if necessary. Although generic, the existing surrogate modeling techniques cannot be directly applied to the problem of variation propagation modeling of multistage manufacturing processes. The first issue is that surrogate models are often chosen based on their interpolative capability and may not be good predictive models. Therefore, the model might not be accurate if the optimization design space is outside the model training space. In practice, people often use a validation step to address this issue. A comparative study of different model validation techniques by Jin *et al.* (2001) recommends the determination of the R^2 error, relative average absolute error, or relative maximum absolute error at untried design sites as a basis for validating the surrogate model. The second issue comes from the fact that a typical manufacturing process (even a single-stage process) has a very large number of parameters to be simultaneously optimized. During the design of a single assembly stage for instance, decisions will be made on the location of numerous locators, leading to the simultaneous optimization of up to 36 parameters (the three coordinates of the three-dimensions of up to 12 locators) with complicated interactions. According to the literature, different surrogate models behave differently in the presence of such complex systems (Chen *et al.*, 2006). While a response surface model does not behave well with such large systems, a NN would require too many simulation runs to train the network, and an ordinary Kriging model may provide a good fit but poor extrapolative power. As a result, these issues need to be taken into consideration in order to establish a successful surrogate model for variation propagation in multistage manufacturing processes.

In this article, we utilize the knowledge drawn from the analytical modeling of multistage assembly processes to derive a general structure of the relationship between the fixture layout and product dimensional quality. Utilizing this physical knowledge, we propose the use of a non-linear regression approach to determine the surrogate model based on rational approximants, which is specified as the ratio of two polynomials, of the complicated relationship between inputs and outputs. Then, the performance of the proposed methodology is compared with ordinary and first-order universal Kriging models in a comprehensive case study.

This paper is organized as follows. The problem formulation is introduced in Section 2. Then, the detailed derivation for the surrogate model of the single part location problem is presented in Section 3. The surrogate model for multistage assembly processes is discussed in Section 4. A comprehensive case study that illustrates the methodology is given in Section 5. Finally, conclusions and some discussions of the application of the proposed methodology are presented in Section 6.

2. Problem formulation

In general, assemblies can be classified into two types: Type I assemblies, and Type II assemblies. In Type I assemblies, the parts are put together at their prefabricated mating features. On the other hand, Type II assemblies need external fixtures to fix the locations of the parts in space during the assembly operation (Whitney, 2004). In practice, an assembly operation may involve both mating features and external fixtures as shown in Fig. 1. In this figure, a solid line is used to represent the positions of the parts constrained by the fixture locators as intended in the design nominal, and a dashed line is used to represent the actual positions of the parts and fixture elements. Figure 1(b) shows the final product as designed. The assembly process is as follows: Part 1 is first located on the fixture and constrained by pins L_1, L_2, L_3 and by a four-way pin L_4 and a two-way pin L_5. Then, Part 2 is located by mating feature f_2 from Part 2 with feature f_1 from Part 1 and by a four-way pin L_9 and a two-way pin L_{10}. The two parts are welded together by joining their mating features, which corresponds to permanently fixturing the second part onto the first part. Nevertheless, the mating of f_1 and f_2 can be represented by the action of locating Part 2 on three locators (L_6, L_7 and L_8) attached to Part 1 on mating feature f_1. Such locators will be referred to as *mating* locators in the remainder of the paper. If Pin L_4's position or diameter deviates from design nominal, then Part 1 will not be in its design nominal position, as shown in Fig. 1(c). After joining Part 1 and Part 2, the final part's dimensions will deviate from the designed nominal values. In this example, the mislocation of the locating pin can be manifested by mean shift or variance change in the dimensional measurement, for example due to errors in the design or the setup of the fixture. In the case of a mean shift error, the error can be compensated by process adjustment, for example by lowering Pin L_9 to align Part 2 with Part 1. The variance change error can be caused by a variation of the location of Pin L_4, for example due to pin wear-out or the excessive looseness of that pin. The variance change

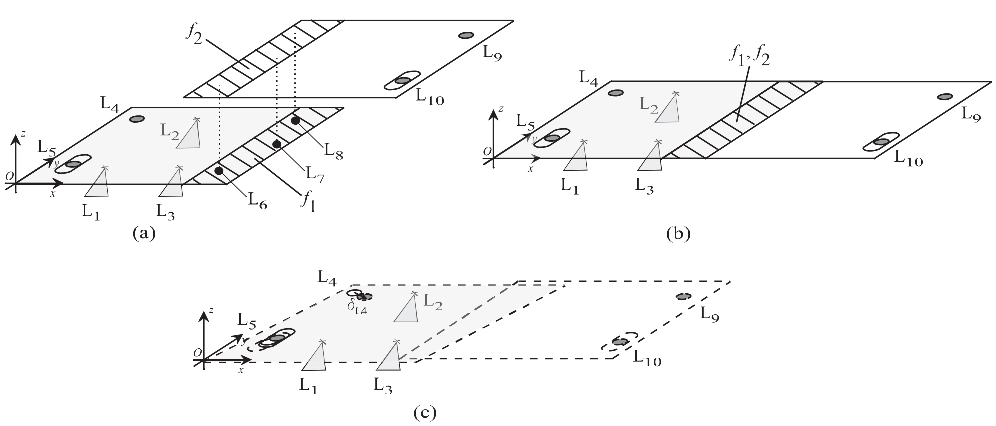

Fig. 1. Illustration of an operation involving both Type I and Type II assemblies.

error causing an excessive variation of process parameters, called "variation source," cannot be easily compensated for in most cases, unless an adaptive assembly capability exists. Also, errors in Part 1 will clearly affect Part 2 and the final dimensional quality. In this article, we provide a modeling methodology for variation propagation in multistage assembly processes, where interactions between parts exist at any given assembly stage.

The task of modeling variation propagation in a multistage assembly process involves the determination of the relationship between process and product parameters (such as the locator positions, the tolerances on locator variations, the geometry of the mating features, the assembly sequence, etc.) and the product dimensional quality (such as the key dimension of the part or the location of specific measurement points). Previous research work has shown that the relationship between locator position variations and the part dimensional quality can be well approximated as linear when the tolerances are small compared to the size of the part. Furthermore, under this condition, the variation at a given stage was shown to impact the variation at subsequent stages in a linear manner (Jin and Shi, 1999). However, the relationship between other design parameters (e.g., the nominal position of fixture locations) and the final production dimensional quality has not been systematically studied. In order to enable the optimization of those process and product parameters, an easy-to-compute expression of their relationships with product dimensional quality has to be known so that they can be part of an optimization routine.

This problem can be expressed mathematically. Figure 2 illustrates a general fixturing system for a single part. In this system, there are n locators (either physical or mating locators) denoted as $L_1, L_2, \ldots, L_n$. The position of the part in space can be determined by the relative location of the part coordinate system ($O'X'Y'Z'$) in the world coordinate system ($OXYZ$). This relative location is represented by a homogeneous transformation matrix between $O'X'Y'Z'$ and $OXYZ$ that involves six independent variables (i.e., three for translation and three for rotation). If we define $\mathbf{t}_i$ as the nominal coordinates of L_i, $\mathbf{v}_i$ as the small deviations of L_i from $\mathbf{t}_i$, and $\mathbf{q}$ as a 6 by 1 vector representing the deviations of $O'X'Y'Z'$ from it is nominal location, then $\mathbf{q}$ represents the dimensional accuracy of the part location and is a complex function of $\mathbf{t}_i$ and $\mathbf{v}_i$, $i = 1, \ldots, n$.

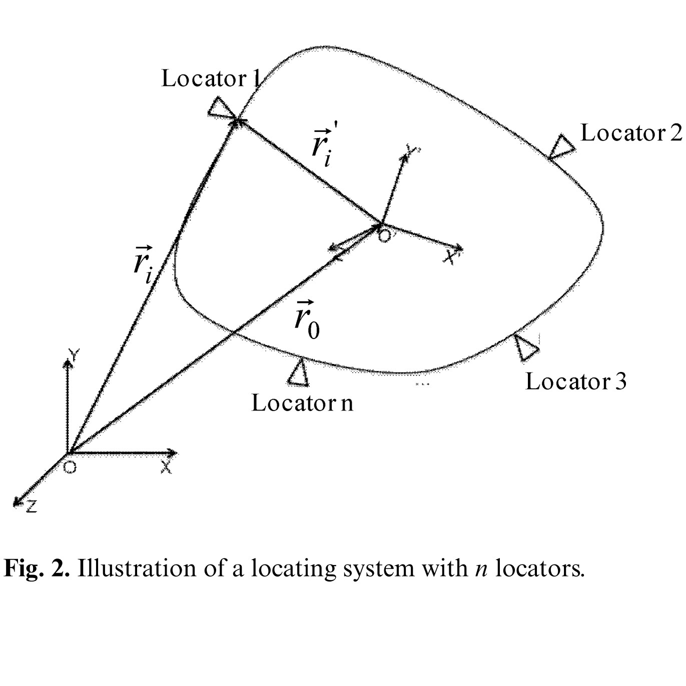

Fig. 2. Illustration of a locating system with n locators.

It is very difficult, if not impossible to derive the exact analytical function between $\mathbf{q}$ and $\mathbf{t}_i$ and $\mathbf{v}_i$, $i = 1, \ldots, n$. However, in practice, computer simulation can be used to compute $\mathbf{q}$ for a specific values of $\mathbf{t}_i$ and $\mathbf{v}_i$, $i = 1, \ldots, n$. In other words, if we specify m process design settings denoted by $\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_m$, where $\mathbf{s}_j = [\mathbf{t}_1^{\mathrm{T}}(j)\ \mathbf{t}_2^{\mathrm{T}}(j) \cdots \mathbf{t}_n^{\mathrm{T}}(j)\mathbf{v}_1^{\mathrm{T}}(j)\mathbf{v}_2^{\mathrm{T}}(j) \cdots \mathbf{v}_n^{\mathrm{T}}(j)]^{\mathrm{T}}$ is a $6n\times 1$ vector representing the jth design setting, then for each design $\mathbf{s}_j$, we can get the part deviation vector $\mathbf{q}_j$ through simulation. Our goal is to find a surrogate function through the available data $(\mathbf{s}_j, \mathbf{q}_j)$, $j = 1, \ldots, m$ to approximate the relationship between $\mathbf{q}$ and corresponding locator position and deviations $\mathbf{s}$:

$$\mathbf{q} = \mathbf{f}(\mathbf{s}, \boldsymbol{\gamma}) + \varepsilon, \tag{1}$$

where $\boldsymbol{\gamma}$ represents the parameters in the model and ε represents the modeling errors. In the following sections, we will describe in detail the methodology of obtaining the surrogate model for a single part, and how to link the single part models together for a multistage assembly process.

3. Surrogate model for the single part locating problem

3.1. *Determination of the structure of function* $\mathbf{f}(\mathbf{s},\boldsymbol{\gamma})$

Instead of assuming a very general model structure as that in most other surrogate modeling techniques, we will conduct an engineering analysis of the problem first and try to identify a specific model structure that well approximates the underlying true function.

Again consider the situation in Fig. 2. The locators should be in the tangent plane of the contact point. Thus, we have the following constraint for the ith locator $\mathbf{n}_i'^{\mathrm{T}}\mathbf{A}^{\mathrm{T}}(\mathbf{r}_i - \mathbf{r}_0) - \mathbf{n}_i'^{\mathrm{T}}\mathbf{r}'_i = 0$, where $\mathbf{n}_i'^{\mathrm{T}}$ is the normal vector at the contact point and $\mathbf{A}$ is the coordinate transformation matrix (Cai *et al.*, 1997). If we let $\mathbf{R} = [\mathbf{r}_1^{\mathrm{T}}, \mathbf{r}_2^{\mathrm{T}}, \cdots, \mathbf{r}_n^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathbf{q}_0$ denote the spatial location of the workpiece, we can combine all the constraint equations for each locator and obtain the complete set of constraint equations as $\boldsymbol{\Phi}(\mathbf{q}_0, \mathbf{R}) = \mathbf{0}$. This constraint equation guarantees that the locators are in contact with the workpiece. If there is an infinitesimal change in $\mathbf{R}$ (denoted as $\boldsymbol{\delta}\mathbf{R}$), then there should also be a corresponding change in $\mathbf{q}_0$ (denoted as $\boldsymbol{\delta}\mathbf{q}_0$). The relationship between these infinitesimal changes can be obtained through a variational analysis of the constraint equation, which is given as $\boldsymbol{\Phi}_{\mathbf{q}_0} \times \boldsymbol{\delta}\mathbf{q}_0 + \boldsymbol{\Phi}_{\mathbf{R}} \cdot \boldsymbol{\delta}\mathbf{R} = \mathbf{0}$ in Cai *et al.* (1997), where $\boldsymbol{\Phi}_{\mathbf{q}_0}$ is the Jacobian matrix $\partial\boldsymbol{\Phi}/\partial\mathbf{q}_0$ and $\boldsymbol{\Phi}_{\mathbf{R}}$ is

$\partial \mathbf{\Phi}/\partial \mathbf{R}$. Denote $\boldsymbol{\delta}\mathbf{q}_0$ as $\mathbf{q}$, $\boldsymbol{\delta}\mathbf{R}$ as $\mathbf{U}$, $\mathbf{J}$ as the Jacobian matrix and $\mathbf{N}$ as $\partial \mathbf{\Phi}/\partial \mathbf{R}$ and note that for a fully constraint part, the Jacobian matrix is invertible; we have:

$$\mathbf{q} = -\mathbf{J}^{-1} \times \mathbf{N} \times \mathbf{U}. \tag{2}$$

Comparing Equation (2) with Equation (1), it is clear that $\mathbf{U}$ consists of $\mathbf{v}_i$, $i = 1, \ldots, n$ in $\mathbf{s}$ and $\mathbf{J}$ and $\mathbf{N}$ are non-linear functions of the locator positions $\mathbf{t}_i$, $i = 1, 2, \ldots, n$. For a general fixturing setting, the structures of $\mathbf{J}$ and $\mathbf{N}$ are very complicated. However, for some specific fixture layouts, the structure can be simplified and become tractable. In this paper, we will look at the 3-2-1 fixture layout, which is the most commonly used layout in practice.

In a 3-2-1 fixture layout, three locators constitute the primary datum, two other locators constitute the secondary datum and the last locator constitutes the tertiary datum. Under this layout, we have a 6×18 matrix $\mathbf{N}$ as

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_1^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{n}_2^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{n}_3^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{n}_4^{\mathrm{T}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{n}_5^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{n}_6^{\mathrm{T}} \end{bmatrix}$$

where

$$\mathbf{n}_i = \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \end{bmatrix},$$

is the outgoing norm to the surface of the feature contacting the ith locator and $\mathbf{0} = [0\,0\,0]$. Also the 3-2-1 scheme ensures that $\mathbf{n}_1 = \mathbf{n}_2 = \mathbf{n}_3$ and $\mathbf{n}_4 = \mathbf{n}_5$. The 6×6 matrix $\mathbf{J}$ can also be expressed as $\mathbf{J} = [\mathbf{J}_1 \cdots \mathbf{J}_6]^{\mathrm{T}}$ where

$$\mathbf{J}_i = \big[-\mathbf{n}_i^{\mathrm{T}} (n_{iy} \times t_{iz} - n_{iz} \times t_{iy})\ (n_{iz} \times t_{ix} - n_{ix} \times t_{iz}) \\ (n_{ix} \times t_{iy} - n_{iy} \times t_{ix}) \big]^{\mathrm{T}}, \tag{3}$$

where $\mathbf{t}_i = [t_{ix}\, t_{iy}\, t_{iz}]$ are the coordinates of the ith locator in a reference coordinate system attached to the part. The details of the derivation for Equation (3) can be found in Loose *et al.* (2007).

From Equation (3), we can know that $\mathbf{J}^{-1}$ should be a matrix whose entries are rational polynomial functions of independent variables $\mathbf{t}_i$, $i = 1, 2, \ldots, n$. Here a rational polynomial function refers to a function of the ratio between two polynomial functions. Through a very tedious derivation that can be conducted using symbolic computation software such as the symbolic toolbox of Matlab, a generic form for the elements of $\mathbf{J}^{-1}$ can be derived. Using a block matrix form, we have:

$$\mathbf{J}^{-1} = \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{ccc|c|c|c} & \mathrm{RP}_1 & & \mathrm{RP}_3 & \mathrm{RP}_4 & \mathrm{RP}_5 \\ \hline & \mathrm{RP}_2 & & \multicolumn{2}{c|}{\mathrm{RP}_6} & \mathrm{RP}_7 \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}. \tag{4}$$

In Equation (4), RP_i is a block matrix of proper dimensions whose elements have the same order rational polynomial form and the same set of variables. Specifically, we have:

$$\begin{aligned} \mathrm{RP}_1 &= \left[\frac{N^3(\mathbf{t}_2, \mathbf{t}_3, \mathbf{t}_4, \mathbf{t}_5, \mathbf{t}_6)}{D^3(\mathbf{t}_2, \mathbf{t}_3, \mathbf{t}_4, \mathbf{t}_5, \mathbf{t}_6)}\right]_{3\times 3}, \\ \mathrm{RP}_2 &= \left[\frac{N^2(\mathbf{t}_2, \mathbf{t}_3, \mathbf{t}_4, \mathbf{t}_5)}{D^3(\mathbf{t}_2, \mathbf{t}_3, \mathbf{t}_4, \mathbf{t}_5, \mathbf{t}_6)}\right]_{3\times 3}, \quad \mathrm{RP}_3 = \left[\frac{N^1(\mathbf{t}_5, \mathbf{t}_6)}{D^1(\mathbf{t}_4, \mathbf{t}_5)}\right]_{3\times 1} \\ \mathrm{RP}_4 &= \left[\frac{N^1(\mathbf{t}_4, \mathbf{t}_6)}{D^1(\mathbf{t}_4, \mathbf{t}_5)}\right]_{3\times 1}, \quad \mathrm{RP}_5 = \left[\frac{N^0}{1}\right]_{3\times 1}, \\ \mathrm{RP}_6 &= \left[\frac{N^0}{D^1(\mathbf{t}_4, \mathbf{t}_5)}\right]_{3\times 2}, \quad \mathrm{RP}_7 = [0]_{3\times 1}, \end{aligned} \tag{5}$$

where $N^r(\mathbf{t}_i)$ and $D^r(\mathbf{t}_i)$ are polynomials containing the terms up to the rth order of $\mathbf{t}_i$. Furthermore, it only contains each component of $\mathbf{t}_i$ at its first order and each term does not contain any interaction with the multiplication of coordinates of the same locator. For example, a possible set of terms in $N^2(\mathbf{t}_4, \mathbf{t}_6)$ is: t_{4x}, t_{4y}, t_{4z}, t_{6x}, t_{6y}, t_{6z}, $t_{4x} \cdot t_{6x}$, $t_{4x} \cdot t_{6y}$, $t_{4x} \cdot t_{6z}$, $t_{4y} \cdot t_{6x}$, $t_{4y} \cdot t_{6y}$, $t_{4y} \cdot t_{6z}$, $t_{4z} \cdot t_{6x}$, $t_{4z} \cdot t_{6y}$, $t_{4z} \cdot t_{6z}$.

Define the ith row of the matrix $-\mathbf{J}^{-1} \times \mathbf{N}$ as $\mathbf{a}_i^{\mathrm{T}}$, then we have $-\mathbf{J}^{-1} \times \mathbf{N} = [\mathbf{a}_1\ \mathbf{a}_2 \cdots \mathbf{a}_6]^{\mathrm{T}}$. Correspondingly, for the ith element of $\mathbf{q}$, denoted by q_i, we have $q_i = \mathbf{a}_i^{\mathrm{T}} \times \mathbf{U}$. From the structure of $\mathbf{J}^{-1}$ and $\mathbf{N}$, it is easy to find that the jth element of $\mathbf{a}_i$, denoted by $\mathbf{a}_i^j$ can be expressed as

$$\mathbf{a}_i^j = (\mathbf{J}^{-1})_{i,k} \times (\mathbf{n}_k)_l \ \text{ where } k = \lceil j/3 \rceil,\ \ l = j - 3k \\ \text{for } j = 1, \ldots, 18.$$

In this formula, $\lceil f \rceil$ means the smallest integer that is not smaller than f. In theory, $\mathbf{n}_i$ is a function of the location of the locators and the part geometry. However, in most practical situations, $\mathbf{n}_i$ is constant in a given design space. Therefore, we assume $\mathbf{n}_i$ is constant in this paper for the sake of simplicity and $\mathbf{a}_i^j$ is a rational polynomial function times a constant value. From these results, we have $q_i = \sum_{j=1}^{18} \mathbf{a}_i^j \times U^j$, where U^j is the jth element of $\mathbf{U}$. By letting $\mathbf{U} = \tau \times \mathbf{e}_j$ in the numerical simulation, we get $q_i = \tau \times \mathbf{a}_i^j$, where $\mathbf{e}_j = (0 \cdots 1 \cdots 0)^{\mathrm{T}}$ is a zero vector with the jth element being one and τ is a constant scalar representing the magnitude of $\mathbf{U}$. This provides us with an opportunity to estimate each element of $\mathbf{a}_i$ separately. Expressing it in a rational polynomial form, we have:

$$\mathbf{a}_i^j = \frac{\mathbf{N}_{i,j}(\mathbf{s})}{1+\mathbf{D}_{i,j}(\mathbf{s})}, \tag{6}$$

where $\mathbf{N}_{i,j}(\mathbf{s})$ and $\mathbf{D}_{i,j}(\mathbf{s})$ are the polynomial functions of the locator positions of the numerator and the denominator of the rational function $\mathbf{a}_i^j$, respectively. For instance, if $i = 1$ and $j = 5$, then:

$$\mathbf{a}_1^5 = \frac{N^3(\mathbf{t}_2, \mathbf{t}_3, \mathbf{t}_4, \mathbf{t}_5, \mathbf{t}_6)}{1 + D^3(\mathbf{t}_2, \mathbf{t}_3, \mathbf{t}_4, \mathbf{t}_5, \mathbf{t}_6)}.$$

It is worth pointing out that Equation (6) is a linearized approximation result that is obtained based on the variational analysis as in Equation (2). The true function $g_i^j(\mathbf{s})$, whose value can be obtained through simulation, can be expressed as

$$g_i^j(\mathbf{s}) = \frac{\mathbf{N}_{i,j}(\mathbf{s})}{1+\mathbf{D}_{i,j}(\mathbf{s})} + \varepsilon = \mathbf{a}_i^j(\mathbf{s}, \boldsymbol{\beta}) + \varepsilon, \tag{7}$$

where $\boldsymbol{\beta}$ is the coefficients of the numerator and denominator polynomial functions and ε represents the error between the true model and our surrogate model. It needs to be pointed out that in most cases, the structure of the surrogate model is different from that of the underlying true model. In fact, not knowing the underlying model is the key reason for building a surrogate model. Due to this difference, there is always a systematic error between the true model and the surrogate model. To handle this unavoidable error, people often add an error term in the surrogate model (i.e., ε in Equation (7)) and assume that the error term is random with mean-zero and variance σ^2. By looking at the variance of the error term, we can evaluate accuracy of the surrogate model.

3.2. *Parameter estimation for rational functions*

Based on Equation (7), the problem of surrogate model fitting can be defined as follows: knowing the values of $g_i^j(\mathbf{s})$, U^j at m process design sites $\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_m$ from the computer simulations, identify the optimal values of $\boldsymbol{\beta}$ and the corresponding value of σ^2. The solution to this question can be obtained by minimizing the least squares criterion, that is

$$\min_{\boldsymbol{\beta}} \sum_{l=1}^{m} \left[\mathbf{a}_i^j(\mathbf{s}_l, \boldsymbol{\beta}) - g_i^j(\mathbf{s}_l)\right]^2. \tag{8}$$

This optimization problem is a non-linear least square problem, and can be solved numerically using some generic optimization algorithms, for example the Levenberg–Marquardt method or the Gauss–Newton method. Especially when the starting point is close to the optimum, the convergence will be fast. The variance of ε can be estimated by

$$\hat{\sigma}^2 = \frac{\sum_{l=1}^{m} \left[\mathbf{a}_i^j(\mathbf{s}_l, \boldsymbol{\beta}) - g_i^j(\mathbf{s}_l)\right]^2}{(m-1)}. \tag{9}$$

It needs to be mentioned that the dimensions of $\boldsymbol{\beta}$ are quite high in most cases. For example, in a typical situation as illustrated in the case study, the polynomial functions include up to 52 terms. Thus, the optimization problem in Equation (8) is very computationally expensive or even fails to converge if an arbitrary starting point is given. Therefore, it is critical to determine a good initial estimate of $\boldsymbol{\beta}$ to use as a starting point for the optimization algorithm. We will describe a procedure to get the initial value through a solution to an approximate linear least square problem as follows.

By multiplying both sides of Equation (7) by $1 + \mathbf{D}_{i,j}(s)$ and rearranging the terms, it is possible to write that:

$$g_i^j(\mathbf{s}) = U^j \times \mathbf{N}_{i,j}(\mathbf{s}) - g_i^j(\mathbf{s}) \times \mathbf{D}_{i,j}(\mathbf{s}) + \varepsilon', \tag{10}$$

where ε' is $(1 + \mathbf{D}_{i,j}(\mathbf{s})) \times \varepsilon$. We can instead consider the following linear least square problem as

$$\min_{\boldsymbol{\beta}} \sum_{l=1}^{m} \left[g_i^j(\mathbf{s}_l) - U^j \times \mathbf{N}_{i,j}(\mathbf{s}_l) + g_i^j(\mathbf{s}_l) \times \mathbf{D}_{i,j}(\mathbf{s}_l)\right]^2, \tag{11}$$

which can be solved efficiently. Denote the ordinary solution to this linear least square problem as $\tilde{\boldsymbol{\beta}}$. It needs to be pointed out that because we cannot assume the error term ε' in Equation (10) follows a Gaussian distribution with zero mean, $\tilde{\boldsymbol{\beta}}$ is not the optimal solution for Equation (8). However, our experience shows that $\tilde{\boldsymbol{\beta}}$ provides a good initial point for the problem shown in Equation (8). Based on these initial estimates $\tilde{\boldsymbol{\beta}}$, the set of optimal parameters $\tilde{\boldsymbol{\beta}}$ for Equation (8) can be obtained through non-linear optimization. By repeating the above procedure for each j from one to 18, all the rational terms for q_i can be identified and then the overall surrogate model for q_i can be established as

$$q_i = \sum_{j=1}^{18} \mathbf{a}_i^j(\mathbf{s}) \times U^j = \sum_{j=1}^{18} \frac{\mathbf{N}_{i,j}(\mathbf{s})}{1+\mathbf{D}_{i,j}(\mathbf{s})} \times U^j.$$

The same procedure can be repeated for each i from one to six and eventually a complete surrogate model can be established for $\mathbf{q} = [q_1, q_2 \cdots q_6]^{\mathrm{T}}$.

3.3. *Model validation*

The surrogate model is validated by checking the differences between the surrogate model outputs and the simulation outputs at some new process design sites that are not used in the model fitting. The overall departure of the surrogate model from the true simulation model can be evaluated by the R^2 value as given below:

$$R^2 = 1 - \frac{[\mathbf{y} - \hat{\mathbf{y}}]^{\mathrm{T}} \cdot [\mathbf{y} - \hat{\mathbf{y}}]}{[\mathbf{y} - \bar{y} \cdot \mathbf{1}]^{\mathrm{T}} \cdot [\mathbf{y} - \bar{y} \cdot \mathbf{1}]} \tag{12}$$

where $\mathbf{y}$ is a 6 $m \times 1$ vector of the true output from the simulation software at m new process design sites, $\bar{y}$ is the average of the simulation software output; $\mathbf{1}$ is a column vector containing $6m$ ones and $\hat{\mathbf{y}}$ is a $6m \times 1$ vector of the predicted output from the fitted model. A larger value of R^2 means a smaller difference between true and predicted values and therefore a surrogate model is more appropriate for prediction.

4. Surrogate model for multiple stages

In Section 3, we describe the methodology to obtain the surrogate model for the locating error of a single part under a 3-2-1 fixture setting. Because an assembly process includes multiple parts at multiple stages and deviations of the locators for a part can propagate to other parts, it is necessary to link the surrogate models for different parts at multiple stages together. In this section, we will describe the procedure to link surrogate models from successive stages together.

In the assembly process as illustrated in Fig. 1, the deviation of Part 1 due to a deviation of its locators will influence the deviation of Part 2 through the mating locators as would a physical fixture for Part 2. The methodology presented in Section 3 allows the determination of the deviation of a single part given that part's fixture layout. In order to link the surrogate models, the propagated errors due to the mating of two parts have to be combined with the local fixture errors of each part. In other words, the dimensional deviation of Part 2 needs to be determined as a function of the dimensional deviation of Part 1. This can be achieved by the following results.

In the assembly operation shown in Fig. 1, if we define ${}^0\mathrm{P}_1$, ${}^0\mathrm{P}_2$ and ${}^0\mathrm{P}_3$ as the nominal mating locators for the assembly of Parts 1 and 2 and $\mathbf{P}_i = [P_{ix}\ P_{iy}\ P_{iz}]^{\mathrm{T}}$ as the nominal coordinates of point ${}^0\mathrm{P}_i$ in the reference coordinate system attached to the part, then the dimensional deviation of the mating point i, denoted as $\mathbf{dP}_i$, is determined as

$$\mathbf{dP}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & P_{iz} & -P_{iy} \\ 0 & 1 & 0 & -P_{iz} & 0 & P_{ix} \\ 0 & 0 & 1 & P_{iy} & -P_{ix} & 0 \end{bmatrix} \times \mathbf{q}_{\mathrm{part1}} \tag{13}$$

where $\mathbf{q}_{\mathrm{part\,1}}$ represents the dimensional deviation of Part 1 in the previous stage, determined following the procedure presented in Section 3. The proof of Equation (13) can be illustrated by Fig. 3.

In Fig. 3, ^{0}RCS represents a reference coordinate system attached to Part 1 under nominal conditions. In this coordinate system, the coordinates of point ${}^0\mathrm{P}_i$ are known as $\mathbf{P}_i$. In the presence of a deviation of Part 1 due to an error of its fixture, as illustrated in Fig. 1, the reference coordinate system attached to the part is in its true location (RCS in Fig. 3). Similarly, point ${}^0\mathrm{P}_i$ will be deviated and its true position is P_i, with coordinate $\mathbf{P}''_i$. The deviation of Part 1 $\mathbf{q}_{\mathrm{part\,1}}$ mathematically expresses the transformation from ^{0}RCS to RCS: Defining q_i as the ith component of $\mathbf{q}_{\mathrm{part\,1}}$, the deviation from ^{0}RCS to RCS can be expressed in the form of a Homogeneous Transformation Matrix (Paul, 1981) $\mathbf{H}_{\mathrm{q}}$ as illustrated in Fig. 3:

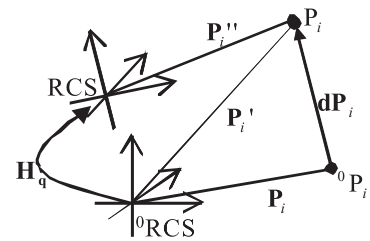

Fig. 3. Illustration of the determination of the deviation of a point given $\mathbf{q}_{\mathrm{part\,1}}$.

$$\mathbf{H}_{\mathbf{q}} = \begin{bmatrix} 1 & -q_6 & q_5 & q_1 \\ q_6 & 1 & -q_4 & q_2 \\ -q_5 & q_4 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{14}$$

Using this transformation, the coordinates $\mathbf{P}''_i$ of a point P_i in RCS can be expressed in the ^{0}RCS as $\mathbf{P}'_i$ using: $[\mathbf{P}'^{\mathrm{T}}_i\ 1]^{\mathrm{T}} = \mathbf{H}_{\mathbf{q}} \times [\mathbf{P}''^{\mathrm{T}}_i\ 1]^{\mathrm{T}}$. Noticing the fact that the coordinates $\mathbf{P}_i$ of point ${}^0\mathrm{P}_i$ in the nominal reference coordinate system (^{0}RCS) are the same as the coordinates $\mathbf{P}''_i$ of point P_i in the deviated reference coordinate system (RCS), since they represent the same physical point on Part 1, e.g., $\mathbf{P}''_i = \mathbf{P}_i$, the deviation $\mathbf{dP}_i = \mathbf{P}'_i - \mathbf{P}_i$ of point ${}^0\mathrm{P}_i$ can be calculated by: $[\mathbf{dP}^{\mathrm{T}}_i\ 1]^{\mathrm{T}} = (\mathbf{H}_{\mathbf{q}} - \mathbf{I}_4) \times [\mathbf{P}^{\mathrm{T}}_i\ 1]^{\mathrm{T}}$. Thus, after some simple manipulations of the matrices, Equation (13) can be obtained.

The deviation $\mathbf{dP}_i$ for $i = 1, \ldots, 3$, gives the deviations of the mating locators for Part 2. After transforming $\mathbf{dP}_i$ into the reference coordinate system of Part 2, we can put them in the vector $\mathbf{U}$ of Part 2. Together with the deviations of the physical locator for Part 2, we can obtain the locator deviation vector $\mathbf{U}$. In this way, the surrogate models for Part 1 and Part 2 can be linked together. This process can be repeated for other parts and eventually models for multiple parts at multiple stages can be linked together.

5. Case studies

5.1. *Process description*

An industrial case study from a simplified automotive door assembly process including two stages (Fig. 4) is used to

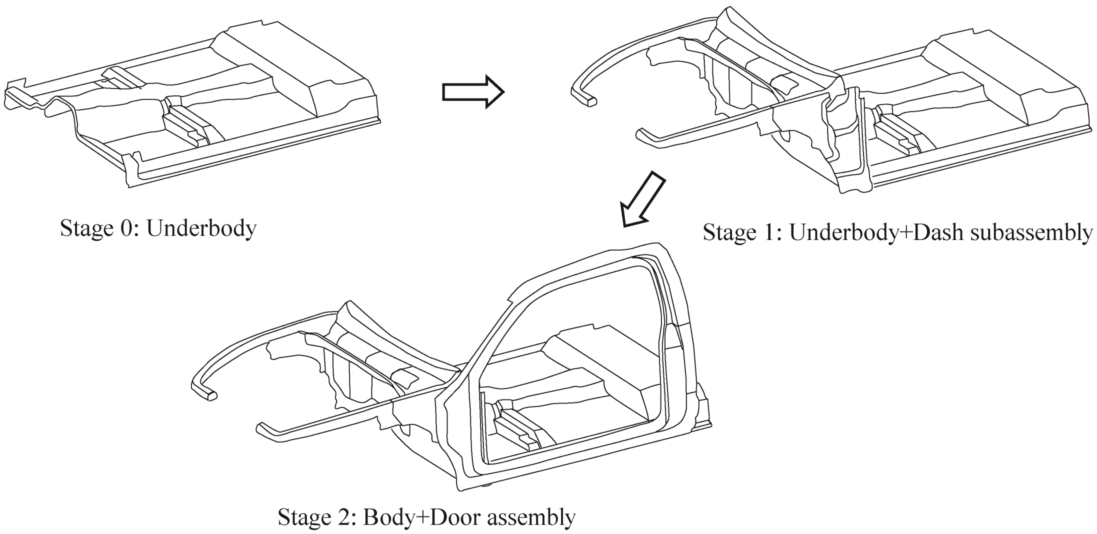

Fig. 4. Two-step simplified door assembly process.

demonstrate the proposed method. The variation propagation simulation output is obtained using 3DCS, a variation simulation software widely used in industry for tolerancing management purposes. The assembly process under consideration consists of two stages and contains three parts. In the first stage, the dash subassembly is assembled with the underbody subassembly and the car body is obtained; in the second stage, the left door is assembled with the body. Figure 4 shows the subassemblies at the end of each station.

In this process, there are 18 locators (12 physical locators and six mating locators). A possible location for all locators as well as the quality measurements of interest (points P_{0_u}, P_{0_da} and P_{0_do}) are shown in Fig. 5. The 18 locators under study imply the possibility to model up to 72 different parameters. In our case study, only 36 parameters are considered, chosen based on physical insights about the process and corresponding to: (i) the positions of locators L_1, L_2 and L_3 in the x and y directions at Stage 1; (ii) the positions of locators L_{13}, L_{14} and L_{15} in the x and z directions at Stage 2; (iii) the positions of locators L_{16}, L_{17} and L_{18} in the x and z directions at Stage 2; (iv) the deviations of locators L_{12}, L_6 and L_{18} in the x-direction; (v) the deviations of locators L_4, L_5, L_{10}, L_{11}, L_{13}, L_{14} and L_{15} in the y-direction; and (vi) the deviations of locators L_1, L_2, L_3, L_7, L_8, L_9, L_{16} and L_{17} in the z-direction.

The design space is defined as follows: the range of interest for the deviations of the locators is $\pm$ 0.5 mm and the number of samples in both fitting and validation is $m = 100$. The nominal location and ranges of interest for the positions of the locators are given in Table 1.

In this assembly process, three product quality characteristics, which are dimensional characteristics that are critical

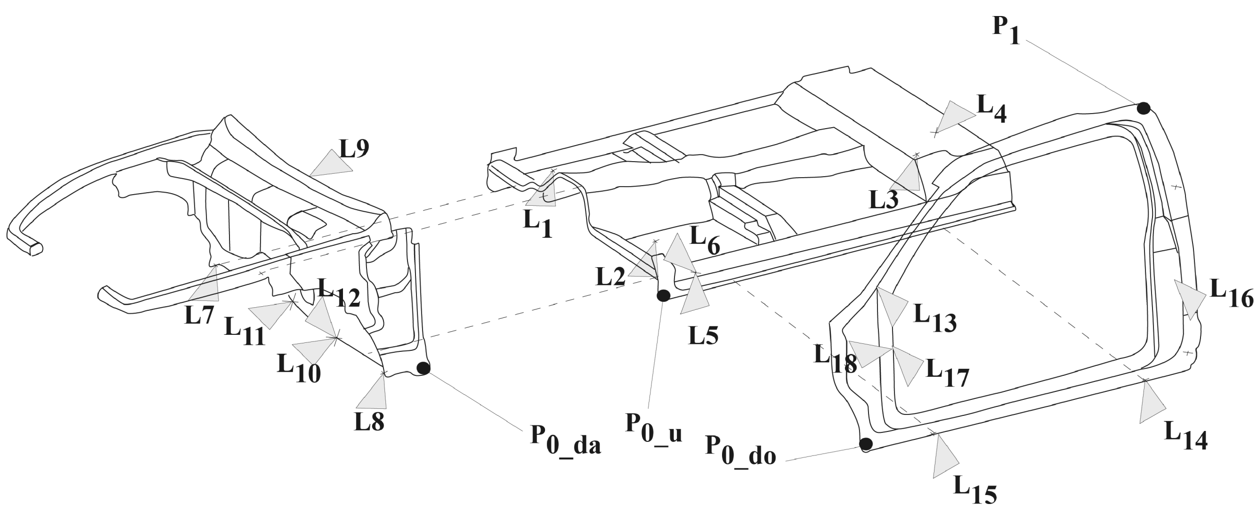

Fig. 5. Process parameters and measurements on the part.

Table 1. Nominal location and ranges of interest for the design space

Locator	Location (mm) [x,y,z]	Range (mm) $\pm\frac{(x_{max}-x_{min})}{2}, \pm\frac{(y_{max}-y_{min})}{2}, \pm\frac{(z_{max}-z_{min})}{2}$
L_1	[140,40,0]	±50, ±50, ±0
L_2	[140,150,0]	±50, ±50, ±0
L_3	[10,150,0]	±50, ±50, ±0
L_4	[10,160,0]	—
L_5	[140,190,0]	—
L_6	[140,190,0]	—
L_7	[220,0,50]	—
L_8	[220,200,50]	—
L_9	[130,100,100]	—
L_{10}	[200,150,50]	—
L_{11}	[200,50,50]	—
L_{12}	[200,150,50]	—
L_{13}	[150,0,50]	±20, ±0, ±20
L_{14}	[30,0,5]	±20, ±0, ±20
L_{15}	[120,0,5]	±20, ±0, ±20
L_{16}	[5,200,40]	±20, ±0, ±20
L_{17}	[150,200,40]	±20, ±0, ±20
L_{18}	[150,200,40]	±20, ±0, ±20

to the fitness of the final assembly, are of interest: (i) the deviation of point P_{0_u} on the underbody in the z direction; (ii) the deviation of point P_{0_da} on the dash subassembly in the x, y and z directions; and (iii) the deviation of points P_{0_do} on the door in the y-direction. All points are defined at the origin of the world coordinate system under nominal conditions. These product quality characteristics are observed in practice as they are critical for the subsequent assemblies of the hood, fenders and roof.

By applying the proposed methodology, the surrogate models describing the relationship between locator positions, deviations and product quality characteristics can be obtained for this two-stage assembly process.

5.2. *Simulation setup for the case study*

In order to evaluate the performance of the proposed methodology, three models will be fitted with the same data generated using 3DCS and their performances in prediction will be compared.

The training dataset is generated using the Latin Hypercube Sampling (LHS) method. LHS is selected because space filling designs such as LHS are very effective for computer experiments (Chen *et al.*, 2006). Since some of the process design sites may not be physically relevant (e.g., three locators aligned on a line or very close to one another), a step to eliminate physically meaningless designs is necessary. In this case study, the relative positions of the locators are constrained by ensuring that the angle between any two vectors from three locators is between 20° and 160°.

The first model is based on the proposed methodology. The other models are based on the popular Kriging models fitted using the available DACE toolbox (Lophaven *et al.*, 2002) for Matlab®. In a Kriging surrogate, an interpolative model is used to describe the relationship between input parameters and output parameters. The model is assumed to be the sum of a regression function and of the realization of a random process with mean $\mathbf{0}$ and covariance $\mathbf{\Sigma}$. The form of the covariance can be chosen from different distributions and will be assumed to be Gaussian in this paper, e.g., the (i, j)th element of $\mathbf{\Sigma}$ is: $\sigma_{ij} = \sigma^2 \times R(\mathbf{s}_i, \mathbf{s}_j)$ with $R(\cdot\ ,\cdot)$ defined as the Gaussian correlation function between two sample points such that:

$$R(\mathbf{s}_i, \mathbf{s}_j) = \prod\nolimits_{k=1}^{n} e^{-\theta(s_{ki}-s_{kj})^2},$$

and n is the number of independent variables. The parameters θ and σ^2 of the covariance are estimated by minimizing the model mean square error. In this case study, the performances of two regression functions (zeroth-order polynomial and first-order polynomial) will be evaluated using the R^2 criterion.

5.3. *Model derivation for the underbody in Stage 1*

In this section, we apply the proposed methodology to obtain a surrogate model for the deviation of the underbody as a function of the location of locators L_1, L_2 and L_3 in the x and y directions (respectively t_{1x}, t_{1y}, t_{2x}, t_{2y}, t_{3x} and t_{3y}) and the deviations (e_1, e_2 and e_3) of locators L_1 through L_6.

In this case study, the expressions of three components of $\mathbf{q}^{\text{underbody}}$ are derived, corresponding to the location and orientation deviations of the underbody as functions of the locators' positions and deviations. These three components correspond physically to the deviation in z (the third element of $\mathbf{q}^{\text{underbody}}$) and the two out-of-plane rotations (the fourth and fifth elements of $\mathbf{q}^{\text{underbody}}$, respectively) of the underbody from its nominal location and orientation and are impacted by the deviations and locations of the locators mentioned above. This set of six variables is selected in this case study to illustrate the effectiveness of the proposed methodology.

For each component q_i of $\mathbf{q}^{\text{underbody}}$ under consideration, and for each deviation U^j under consideration, a surrogate model $q_i^j = \mathbf{a}_i^j(t_{1x}, t_{1y}, t_{2x}, t_{2y}, t_{3x}, t_{3y}) \cdot U^j$ will be fitted using 100 training samples randomly selected within the design space defined in Table 1 using a LHS method. Then, each component $\mathbf{q}_i^j$ obtained will be assembled to form q_i and finally the three components of $\mathbf{q}^{\text{underbody}}$ under consideration are obtained.

Also, based on the same input parameters and the data from the same design sites, a zeroth order and first order Kriging models are fitted. The actual expressions of the fitted surrogate models are extremely long given the large number of variables and they present little insights; therefore, they are omitted here but are available to the reader from the authors.

Table 2. R^2 values (%) for the deviation models of the underbody

Model	q_3	q_4	q_5
Rational	99.99	99.96	99.99
Zeroth-order	65.71	62.08	75.96
First-order	34.61	0.5	67.21

Based on the surrogate models fitted and using 100 newly generated samples, the prediction power of the models is determined by calculating the R^2 value. The new samples are generated based on LHS within the design space as defined in Table 1, and all the variables (t_{1x}, t_{1y}, t_{2x}, t_{2y}, t_{3x}, t_{3y}, U^1, U^2 and U^3) are varying simultaneously in the design. The results are gathered in Table 2.

From the table, it is clear that the proposed model based on rational approximants outperforms the other models. For the out-of-plane rotation, the performances of the Kriging models are good. This is an expected result, since the underlying model is of the zeroth or first order, as shown in Equation (5).

Also, it is clear from the table that the first-order Kriging model in fact performs worse than the zeroth order Kriging model in this case. This observation corroborates the conclusion made by Welch *et al.* (1992) that using higher-order polynomials does not help obtain better predictions in a Kriging model. The result shows that considering physical knowledge about the process gives great advantage in being able to limit the number of parameters to estimate.

5.4. *Numerical results for the dash subassembly in Stage 1 and door assembly in Stage 2*

5.4.1. *Dash subassembly modeling*

Following the same simulation procedure, surrogate models for the six components of $\mathbf{q}^{\text{dash}}$, describing the location and orientation deviation of the dash subassembly, are obtained as a function of: (i) the positions of locators L_{10} through L_{12}; and (ii) the deviations of mating locators L_7, L_8 and L_9 in the z-direction and physical locators L_{10} through L_{12} in the x and y directions, respectively. Also, given the expression of the deviation $\mathbf{q}^{\text{underbody}}$ of the underbody found in the previous section and given the nominal location of mating locators L_7, L_8 and L_9 defined in Table 1, the expressions of the deviations of locators L_7, L_8 and L_9 in the z-direction, physically critical to determine the influence of the deviation of the underbody on the deviation of the dash subassembly, are obtained using the result obtained in the previous subsection. Then, combining the expressions of the deviations of the mating locators in the components of $\mathbf{q}^{\text{dash}}$ and the expressions of their deviation as functions of the parameters of the underbody assembly, $\mathbf{q}^{\text{dash}}$ is fully determined as a function of: (i) the positions of locators L_1 through L_6; and (ii) the deviations of L_1 through L_{12} in their respective directions as noted earlier; thus, the underbody and the dash subassembly models are linked. Finally, based on the obtained expressions of the components of $\mathbf{q}^{\text{dash}}$, and using the result from the previous section, the deviation of point P_{0_da} in the x, y and z-directions is determined as a function of the location and deviations of the locators used to locate the underbody and dash subassembly.

Table 3. R^2 values (%) for the dash subassembly at stage 1

		Range (mm)				
	Model	*±0.2*	*±0.5*	*±0.75*	*±1*	*±1.5*
$dP_{o\_\text{da}x}$	Rational	77.80	76.26	75.03	73.82	71.46
	Zeroth-order	36.95	50.50	34.58	22.61	7.83
	First-order	6.83	—	—	—	—
$dP_{o\_\text{da}y}$	Rational	75.51	75.94	76.34	76.75	77.63
	Zeroth-order	40.26	44.87	24.86	12.55	39.58
	First-order	—	—	—	—	—
$dP_{o\_\text{da}y}$	Rational	70.85	71.14	71.39	71.65	72.15
	Zeroth-order	10.40	41.37	21.21	9.64	0.2
	First-order	—	—	—	—	—

Based on the obtained model, and using 100 newly generated designs, the prediction performance is evaluated in five scenarios. In each scenario, the variables under consideration are: (i) the locations of locators L_1, L_2 and L_3 in a range as defined in Table 1; and (ii) the deviations of locators L_1, L_2 L_3, L_{10}, L_{11} and L_{12} within a range as specified in Table 3. The performance of the three models is evaluated by calculating the R^2 values and the results are gathered in Table 3.

In the table, a dash replacing a value indicates a negative R^2 value. Mathematically, a negative R^2 value corresponds to a model performing worse for prediction than the simple average of the simulation output and therefore corresponds to an unusable model. Clearly, the results show that the prediction power of the proposed methodology for a single stage is greater than the prediction power of traditional Kriging surrogate models. Also, Table 3 highlights the robustness of the prediction to changes in the ranges of deviations of the locators used to locate the underbody. It is clear that the range of the deviations used to calculate the deviation of the dash subassembly does not impact the prediction power of the proposed methodology. However, the traditional Kriging models are greatly affected by a change in the deviation. This is an expected result, since our method is based on the mathematical relationship between locator positions and deviations derived from physical analysis of the assembly process, whereas traditional Kriging is not.

5.4.2. *Door assembly modeling*

Following a similar model building and simulation procedure as above, the deviation of the door at the second stage ($\mathbf{q}^{\text{door}}$) is determined as a function of $\mathbf{q}^{\text{underbody}}$ (through mating locators L_{14} and L_{15}) and $\mathbf{q}^{\text{dash}}$ (through mating locator L_{13}) and as a function of the positions of locators L_{16}, L_{17} and L_{18} at the second stage. The final surrogate models are obtained by fitting 100 outputs of the simulation software following the proposed methodology. Then, the prediction performances are evaluated using 100 prediction samples given the location ranges in Table 1 for the positions of locators L_1, L_2 and L_3, and two scenarios for the deviations of the locators; in the first scenario, only the deviations of locators L_1, L_2 and L_3 are considered. In the second scenario, the deviations of locators L_{10}, L_{11} and L_{12} are also considered. The performance criteria for the final predictions of the deviations of point $P_{0,do}$ in the y-direction is evaluated by calculating the R^2 criteria. In both simulated cases, the traditional Kriging models failed to predict the deviation of the door at the second stage, due to the propagation of the poor predicted deviations of the mating locators from the two previous stages, when the R^2 values for the proposed methodology were 76.25% and 87.17% for the first and second scenarios respectively. The results show, for Scenario 2, a higher prediction power than Scenario 1. This is an expected result, since locators L_{10}, L_{11} and L_{12} for the dash subassembly have physically a much greater impact on the deviation of the door than locators L_1, L_2 and L_3.

6. Conclusions

In this paper, a novel surrogate modeling technique using rational approximants is proposed for the modeling of variation propagations in multistage assembly processes. The proposed methodology takes advantage of prior knowledge about the relationship between process variables and product dimensional quality. In particular, it takes advantage of the fact that some process parameters have only a linear impact on the product dimensional quality. Furthermore, the known linear relationship between stages is fully utilized to obtain an overall multistage surrogate model. The methodology has been validated using a two-stage case study of an automotive door assembly. The performances are shown to be much better than popular Kriging models. This is mainly due to the fact that the proposed regression function approximates the response surface better in a design space where the function is not stationary, therefore resulting in poor prediction when using traditional Kriging models. To generate the process design sites, a LHS scheme has been utilized. By using a dedicated experimental design, the performance and robustness of the methodology may be further increased. This task will be addressed in future work.

From this paper, we can find that the hybrid modeling framework can be applied wider other than assembly processes. As long as partial physical analysis is feasible and simulation is well developed, this framework (i.e., first-use physical analysis to obtain the structure of the function and then use simulation and statistical estimation to get the parameters of the surrogate function) can be applied. Also, comparing with conventional surrogate modeling techniques that are based purely on simulation data, this hybrid method can reduce the dimension of the problem through the partial physical analysis. Thus, we believe that this hybrid method can handle larger-scale problems.

The developed methodology offers great application potential for engineering design and optimization, from the optimization of fixture layout in a multistage assembly process to the optimization of mating features and datum selections. A significant amount of work needs to be done to address these issues and results will be presented in forthcoming publications.

Acknowledgements

Financial support for this work was provided by the National Science Foundation under grant CMMI-0529327. The authors thank the editor and the referees for their valuable comments and suggestions.

References

Cai, W., Hu, S.J. and Yuan, J.X. (1997) A variational method of robust fixture configuration design for 3-D workpieces. *Transactions of the ASME, Journal of Manufacturing Science and Engineering*, **119**, 593–602.

Ceglarek, D., Huang, W., Zhou, S., Ding, Y, Kumar, R. and Zhou Y. (2004) Time-based competition in manufacturing: stream-of-variation analysis (SOVA) methodology – review. *International Journal of Flexible Manufacturing Systems*, **16**(1), 11–44.

Ceglarek, D. and Shi, J. (1996) Fixture failure diagnosis for the autobody assembly using pattern recognition. *Transactions of the ASME, Journal of Engineering for Industry*, **118**(1), 55–66.

Ceglarek, D. and Shi, J. (1999) Fixture failure diagnosis for sheet metal assembly with consideration of measurement noise. *Transactions of the ASME, Journal of Manufacturing Science and Engineering*, **121**(4), 771–777.

Chen, V.C.P., Tsui, K.L., Barton, R.R. and Meckesheimer, M. (2006) A review on design, modeling and applications of computer experiments. *IIE Transactions*, **38**(4), 273–291.

Ding, Y., Ceglarek, D. and Shi, J. (2000) Modeling and diagnosis of multistage manufacturing processes: part I state space model, in *Proceedings of the 2000 Japan/ USA Symposium on Flexible Automation*, 2000JUSFA-13146, Ann Arbor, MI, pp. 233–240.

Ding, Y., Ceglarek, D. and Shi, J. (2002a) Fault diagnosis of multistage manufacturing processes by using state space approach. *Transactions of the ASME, Journal of Manufacturing Science and Engineering*, **124**(2), 313–322.

Ding, Y., Ceglarek, D. and Shi, J. (2002b) Design evaluation of multi-station assembly processes by using state space approach. *Transactions of the ASME, Journal of Mechanical Design*, **124**(3), 408–418.

Ding, Y., Jin, J., Ceglarek, D. and Shi, J. (2005) Process-oriented tolerancing for multi-station assembly systems. *IIE Transactions*, **37**(6), 493–508.

Ding, Y., Kim, P., Ceglarek, D. and Jin, J. (2003) Optimal sensor distribution for variation diagnosis in multi-station assembly processes. *IEEE Transactions on Robotics and Automation*, **19**(4), 543–556.

Ding, Y., Zhou, S. and Chen Y. (2005) A comparison of process variance estimation methods for in-process dimensional control. *Transactions of the ASME, Journal of Dynamic Systems, Measurement, and Control*, **127**(1), 69–79.

Djurdjanovic, D. and Ni, J. (2001) Linear state space modeling of dimensional machining errors. *Transactions of NAMRI/SME*, **29**, 541–548.

Huang, Q., Zhou, N. and Shi, J. (2000) Stream of variation modeling and diagnosis of multi-station machining processes, in *Proceedings of the 2000 ASME International Mechanical Engineering Congress and Exposition (IMECE'00)*, Orlando, FL, pp. 81–88.

Jin, J. and Shi, J. (1999) State space modeling of sheet metal assembly for dimensional control. *Transactions of the ASME, Journal of Manufacturing Science and Engineering*, **121**(4), 756–762.

Jin, R., Chen, W. and Simpson, T.W. (2001) Comparative studies of metamodeling techniques under multiple modeling criteria. *Structural and Multidisciplinary Optimization*, **23**(1), 1–13.

Khan, A. and Ceglarek, D. (2000) Sensor optimization for fault diagnosis in multi-fixture assembly systems with distributed sensing. *Transactions of the ASME, Journal of Manufacturing Science and Engineering*, **122**(1), 215–226.

Li, Z. and Zhou, S. (2006) Robust method of multiple variation sources identification in manufacturing processes for quality improvement. *Transactions of the ASME, Journal of Manufacturing Science and Engineering*, **128**(1), 326–336.

Li, Z., Zhou, S. and Ding, Y. (2007) Pattern matching for root cause identification of manufacturing processes with consideration of general structured noise. *IIE Transactions*, **39**(3), 251–263.

Loose, J.-P., Zhou, S. and Ceglarek, D. (2007) Kinematic analysis of dimensional variation propagation for multistage machining processes with general fixture layouts. *IEEE Transactions on Automation Science and Engineering*, **4**(2), 141–152.

Lophaven, S.N., Nielsen H.B. and Sondergaard, J. (2002) DACE – a Matlab Kriging toolbox, Report, Technical University of Denmark, Lyngby, Denmark.

Myers, R.H. and Montgomery, D.C. (1995) *Response Surface Methodology: Process and Product Optimization Using Designed Experiments,* Wiley, New York, NY.

Paul, R.P. (1981) *Robot Manipulators: Mathematics, Programming, and Control: The Computer Control of Robot Manipulators,* MIT Press, Cambridge, MA.

Sacks, J., Welch, W.J., Mitchell, T.J. and Wynn, H.P. (1989) Design and analysis of computer experiments. *Statistical Science*, **4**(4), 409–435.

Shiu, B.W., Ceglarek, D. and Shi, J. (1996) Multi-stations sheet metal assembly modeling and diagnostics. *Transactions of NAMRI/SME*, **24**, 199–204.

Simpson, T.W., Mauery, T.M., Korte, J.J. and Mistree, F. (1998) Comparison of response surface and Kriging models for multidisciplinary design optimization, in *Proceedings of the 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Part 1*, St. Louis, MO, pp. 381–391.

Simpson, T.W., Peplinski, J., Koch, P.N. and Allen, J.K. (2001) Metamodels for computer-based engineering design: survey and recommendations. *Engineering with Computers*, **17**(2), 129–150.

Wang, G. and Shan, S. (2007) Review of metamodeling techniques in support of engineering design optimization. *Transactions of the ASME, Journal of Mechanical Design*, **129**(4), 370–380.

Welch, W.J., Buck, R.J., Sacks, J., Wynn, H.P., Mitchell, T.J. and Morris, M.D. (1992) Screening, predicting, and computer experiments. *Technometrics*, **34**(1), 15–25.

Whitney, D.E. (2004) *Mechanical Assemblies: Their Design, Manufacture, and Role in Product Development,* Oxford University Press, Oxford, UK.

Zhong, W., Huang, Y. and Hu, S.J. (2002) Modeling variation propagation in machining systems with different configurations, in *Proceedings of the 2002 ASME International Mechanical Engineering Congress and Exposition (IMECE'02)*, New Orleans, LA, pp. 97–106.

Zhou, S., Huang, Q. and Shi, J. (2003) State space modeling of dimensional variation propagation in multistage machining process using differential motion vectors. *IEEE Transactions on Robotics and Automation*, **19**(2), 296–309.

Biographies

Jean-Philippe Loose received the B.S. degree in industrial engineering from the ENSIACET engineering school of France in 2004 and the M.S. degree and Ph.D. degree in Industrial and Systems engineering from the University of Wisconsin, Madison, in 2004 and 2008, respectively. Currently, he is a Business Analyst in Global Supply Chain Management at Cisco Systems. Dr. Loose is a member of the Society of Mechanical Engineers (SME) and Institute of Operations Research and Management Sciences (INFORMS).

Nan Chen is a Ph.D. student in the Department of Industrial and Systems Engineering at University of Wisconsin Madison. His research interests include statistical modeling and monitoring of manufacturing and service process, service optimization via simulation and discrete event sequence analysis. He is a member of INFORMS, SME.

Shiyu Zhou is an associate professor in the Department of Industrial and Systems Engineering at the University of Wisconsin-Madison. He received his B.S. and M.S. in Mechanical Engineering at University of Science and Technology of China in 1993 and 1996 respectively, and his Master in Industrial Engineering and Ph.D. in Mechanical Engineering at the University of Michigan in 2000. Dr. Zhou's research interests are the in-process quality and productivity improvement methodologies by integrating statistics, system and control theory and engineering knowledge. His research is sponsored by National Science Foundation, Department of Energy, NIST-ATP and industries. He is a recipient of the CAREER Award from the National Science Foundation in 2006. Dr. Zhou is a member of IIE, INFORMS, ASME and SME.