Paleoclassical Electron Heat Transport
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An important new mechanism for radial electron heat transport has recently been identified [1,2]. It is due to a combination of parallel free-streaming and the “paleoclassical” Coulomb collisional processes of magnetic field diffusion and parallel electron heat conduction in low collisionality toroidal plasmas. The key new physical point is that as magnetic field lines diffuse radially [with $D_\| \approx \eta_0/\mu_0 \equiv m_e\nu_e/(\mu_0 n_e e^2) = \nu_e c^2/\omega_p^2$], they carry with them electron heat equilibrated over a long parallel length $L$, which is the minimum of the electron collision length $\lambda_e$ and a maximum effective field line length. Since $L$ is much longer than the poloidal periodicity length $\pi R_0 q$, the radial electron heat diffusivity is $M \sim (L/\pi R_0 q) \gg 1$ times the magnetic field diffusivity: $\chi_e^0 \sim M \nu_e c^2/\omega_p^2$. Subsequent paragraphs: develop the paleoclassical model (I–III), and show how it could explain many features of “anomalous” electron heat diffusion in large aspect ratio tokamaks (IV) and other current-carrying axisymmetric toroidal plasmas (V).

I. Magnetic Field Model, Diffusion. Consider a sheared slab magnetic field: $\mathbf{B} = B_0 \hat{e}_x + \hat{e}_z \times \nabla \psi$, in which $\hat{e}_z$ is along a rational field line, $\psi \equiv B_0 x^2/2L_S$ is the magnetic flux of the field shear, and $1/L_S \approx s/R_0 q$ for $s \equiv q/\sqrt{\pi}$ in a large aspect ratio ($\epsilon \equiv r/R_0 \ll 1$) tokamak. Magnetic shear is caused by a parallel current in the plasma: $J \equiv J_x \hat{e}_x + J_z \hat{e}_z$, where $J_x = \nabla^2 \psi/\mu_0$. The evolution equation for $\psi$ is obtained from a combination of Faraday’s and Ampere’s law using an Ohm’s law that includes electron inertia:

$$
\left(1 - \delta_e^2 \nabla^2 \right) \frac{d\psi}{dt} = \frac{\eta_\parallel}{\mu_0} \nabla^2 \psi - \frac{\partial \Psi}{\partial t}, \quad \text{with} \quad \delta_e \equiv \frac{c}{\omega_p} \quad \text{(em skin depth)} \quad \text{and} \quad \frac{\partial \Psi}{\partial t} \equiv E_x^A.
$$

Here, $E_x^A$ is the inductive electric field; it is a source of magnetic flux (field lines) and balances the magnetic field diffusion to produce an equilibrium $\mathbf{B}$ on the resistive time scale. For $\delta_e^2 \nabla^2 \psi << 1$ (i.e., $\psi > \delta_e$), $\psi$ and hence field lines diffuse with a diffusion coefficient $D_\| \approx \eta_0/\mu_0 \equiv \nu_e \delta_e^2 \sim (\Delta x)^2/\Delta t$, which implies diffusive steps $\Delta x \approx \delta_e$ in a collision time $\Delta t = 1/\nu_e$. For $|x| < \delta_e$ the solution of (1) for $\psi$ is spatially constant and produces no field lines or diffusion of them.

II. Field Line Length. Magnetic flux surfaces are rational or irrational depending on whether $q(r)$ is a ratio of rational numbers (i.e., $m/n$) or not. On rational field lines close on themselves after $n$ poloidal transits. The length of rational field lines in a large aspect ratio tokamak is $2\ell_{m/n} \approx 2\pi R_0 q n$. In a sheared magnetic field the axial length over which a field line diffuse radially (in $|x| > \delta_e$ region) is $[1,2] \ell_\delta \equiv L_S/(\delta_e n q/r)$. Setting $\ell_{m/n} = \ell_\delta$ yields a maximum $n$, typically $\gtrsim 10$, and the maximum length $\ell_{max}$ for diffusing field lines:

$$
n_{\max} = \frac{1}{\sqrt{\pi \delta_e q}}, \quad q' \equiv \frac{dq}{dr} \bigg|_{r_\delta} \implies 2\ell_{\max} \approx 2\pi R_0 q n_{\max}, \quad \text{maximum field line length.} \quad (2)
$$

However, near a low order rational surface [e.g., $q^0 \equiv m^0/n^0 = 3/2$] the relevant $n$ is its value ($n^0$) on that rational surface and the field line length is reduced accordingly (to $2\ell_{n^0} \approx 2\pi R_0 q^0 n^0$). The maximum half-length $L$ over which the electron temperature can be equilibrated is thus

$$
L = \min \{ \ell_{\max}, \lambda_e, \ell_{n^0} \}, \quad \text{equilibration half length.} \quad (3)
$$

III. Paleoclassical Radial Electron Heat Transport. Radial transport of electron heat due to paleoclassical processes is caused by two separate effects. First, and most important, electron heat equilibrated along field lines diffuse radially with the magnetic field diffusivity. To lowest order the electron distribution is a Maxwellian that is constant along the (diffusing) magnetic field lines (to a length of $L$). In the vicinity of each $m/n$ rational surface with $n \leq n_{max}$ this produces a diffusing Maxwellian component [2]: $\partial f_{n/m}/\partial t = \tilde{\nu}_e \delta_e^2 \partial^2 f_M/\partial \nu^2$, in which
\( \nu_e \equiv \frac{\eta_e}{\eta_0} \nu_e \) is an effective electron collision frequency [2] reflecting the parallel neoclassical resistivity in a torus. Summing up these various \( m \), \( n \) contributions using an inverse ballooning transform [3] yields \( \frac{\partial f_M(t)}{\partial t} = \int \frac{df}{(2\pi R_0 q)} e^{ik_i t} \frac{\partial f_M}{\partial t} \approx (L/\pi R_0 q) \frac{\partial f_M}{\partial t} \) in which \( k_i(x) \ell \approx n_0 q \ell / R_0 q < 1 \) for \( \ell \leq \ell_{\text{max}} \). Taking the energy moment of this last result for a Maxwellian that has \( dT_e/dr \neq 0 \) but \( d\phi_e/dr = 0 \) yields an electron energy transport equation \( (3/2) n_e dT_e/dt = \chi_e^{pc} d^2 T_e/dr^2 \), in which the paleoclassical electron heat diffusivity is [2]:

\[
\chi_e^{pc} \approx M \frac{\eta_e^{pc}}{\mu_0}, \quad M \equiv \frac{3}{2} \left( \frac{L}{\pi R_0 q} \right), \quad \frac{\eta_e^{pc}}{\eta_0} \approx \frac{\sqrt{2} + Z}{\sqrt{2} + 13Z/4} + \frac{\mu_e}{\mu_0}, \quad \frac{\eta_0}{\mu_0} \approx \frac{1.4 \times 10^3}{[T_e(eV)]^{3/2}} \text{s}.
\]

Here, \( Z \rightarrow Z_{\text{eff}} \equiv \sum n_i Z_i^2 / n_e \) is the (effective) ion charge, and the parallel electron viscosity effect is \( \mu_e / \nu_e \approx (Z + \sqrt{2} - \ln(1 + \sqrt{2})) f_i / Z f_e \), \( Z = 1 \) in the banana regime [4] with \( f_i \approx 1.46\sqrt{\tau} + O(\epsilon^{3/2}) \) and \( f_e \equiv 1 - f_i \). Note that \( T_e \) relaxes diffusively \( \approx M \) times faster than the magnetic flux \( \psi \) does. The second paleoclassical effect [2] arises from \( \partial \psi / \partial t \neq 0 \) transients.

IV. Large Aspect Ratio Tokamaks. Many salient and mysterious characteristics of “anomalous” radial electron heat diffusion identified over the past three decades [5] can be interpreted in terms of the paleoclassical diffusivity specified by (2)–(4): 1) Magnitude. For a typical ohmically-heated TFTR plasma [6] with \( T_e \approx 1.2 \text{ keV}, n_e \approx 3 \times 10^{19} \text{ m}^{-3}, Z_{\text{eff}} = 2, R_0 \approx 2.55 \text{ m}, q \approx 1.6 \), \( 1/q' = 0.4 \text{ m} \text{ m}^{-1} \text{ at } r/a \approx 0.4/0.8 = 0.5 \), one obtains \( \eta_0 / \mu_0 \approx 0.067 \text{ m}^2 / \text{s}, \eta_e^{pc} / \eta_0 \approx 2.2 \), \( \delta_e \approx 10^{-3} \text{ m}, n_{\text{max}} \approx 11 \), and \( \lambda_e \approx 300 \text{ m} > \pi R_0 q n_{\text{max}} \approx 140 \text{ m} \), so that \( L \approx \pi R_0 q n_{\text{max}}, M = (3/2) n_{\text{max}} \approx 17 \), and the estimated \( \chi_e^{pc} = 2.5 \text{ m}^2 / \text{s} \approx \chi_e^{\exp} \). 2) Radial Variation. In the usually applicable “collisionless paleoclassical regime” \( \lambda_e > \pi R_0 q n_{\text{max}}, \chi_e^{pc} \approx n_e^{1/4}(r) / [q' T_e^2(r)]^{1/2} \) increases significantly as \( T_e \) decreases from the hot center toward the cooler plasma edge. 3) Collisionality Regime. Paleoclassical transport is only applicable in the banana-plateau collisionality regime, where tokamaks invariably operate. 4) Density Scaling. For high density “collisional” plasmas with \( \pi R_0 q < \lambda_e < \pi R_0 q n_{\text{max}}, \chi_e^{pc} \approx (3/2)(\eta_e^{pc} / \eta_0) (v_T e / \pi R_0 q) (c/\omega_p)^2 \), which implies an Alcator-like plasma energy confinement magnitude and scaling [7] \( \tau_{Ee} \propto a^2 / 4 \chi_e^{pc} \propto n_e a^2 R_0 q \) (assuming \( \sqrt{\tau_{T_e}} \propto \text{constant} \). 5) Low Order Rational Surfaces. As indicated in (3), \( L \) and hence \( \chi_e^{pc} \) are much smaller near low order \( (n^2 = 1, 2) \) rational surfaces [8], particularly when \( q \) is near a minimum there [9]. 6) Near Separatrix. On closed field lines just inside the magnetic separatrix, \( q \) and \( q' \) become large, \( n_{\text{max}} \) decreases and \( \chi_e^{pc} \) is significantly reduced [9b].

V. Other Toroidal Plasmas. The general paleoclassical model [2] applies to axisymmetric toroidal plasmas of all types — spherical tokamaks (STs), spheromaks and reversed field pinches (RFPs) — in regions where \( B_p^2 / B_t^2 << 1 \). For \( R_0 \sim 1 \text{ m} \) STs with \( T_e \sim 1 \text{ keV} \) the prediction at \( r/a \sim 0.5 \) is \( \chi_e^{pc} \sim 8 \text{ m}^2 / \text{s} \) (large because for STs \( q' \) is small, and \( \eta_e^{pc} / \eta_0 > 5 \) is large). For quiescent RFP plasmas such as those in MST PPCD plasmas at \( r/a \sim 0.5, \chi_e^{pc} \sim 10 \text{ m}^2 / \text{s} \) (large because \( q < 0.2 \) and hence \( q' \) is small), which is close to \( \chi_e^{\exp} \sim 12 \text{ m}^2 / \text{s} \) [10]. In quasi-symmetric stellarator plasmas there would be no paleoclassical transport if there is no flux-surface-average parallel current in the plasma; however, net parallel currents would induce a \( \chi_e^{pc} \).