PALEOCCLASSICAL ELECTRON HEAT TRANSPORT DETERMINES H-MODE $T_e$ EDGE PROFILE WIDTH?

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Theses:

- Paleoclassical $\chi_e$ is most important, possibly dominant for $T_e \lesssim 1$ keV $\implies$ applicable to DIII-D edge plasmas ($T_{e \text{sep}} \sim 100$ eV up to $\sim 1$ keV).
- In DIII-D plasmas, paleoclassical model predicts the pedestal $T_e \propto n_e^2$ $\implies T_e(\rho)$ determined by edge $n_e(\rho)$ [Mahdavi et al., PoP 10, 3984 (2003)].

Outline:

- Key Hypothesis Of Paleoclassical Model
- Paleoclassical $\chi_e$ Predictions Near A Separatrix
- Pedestal $T_e$ Profile Determined By Edge $n_e$ Profile
- Comparison Of This Model With DIII-D Data
- Summary
Key Hypothesis Of Paleoclassical Model

- Key paleoclassical hypothesis is:
  electron guiding centers diffuse with poloidal magnetic flux, field lines

- Consider \( \vec{F} = m\vec{a} \) for electrons with \( \vec{E} = -\partial\vec{A}/\partial t, \vec{A} = \psi_t \vec{\nabla}\theta - \psi \vec{\nabla}\zeta \):
  \[
  m_e \frac{d\vec{v}}{dt} = -q_e \frac{\partial\vec{A}}{\partial t} + q_e \vec{v} \times \vec{B} \quad \Rightarrow \quad m_e \frac{d(R^2 \zeta)}{dt} = q_e \left( \frac{\partial \psi}{\partial t} + \vec{v} \cdot \vec{\nabla} \psi \right) = q_e \frac{d\psi}{dt}
  \]

- With NO RESISTIVITY, integration over time \( t \) yields constancy of canonical toroidal angular momentum \( p_\zeta \):
  \[
  p_\zeta \equiv R^2 \vec{\nabla} \zeta \cdot (m_e \vec{v} + q_e \vec{A}) = m_e R^2 \zeta - q_e \psi = \text{constant} \approx -q_e \psi \left[ 1 + O(q/\ell) \right]
  \]

- The lowest order distribution function is usually a function of the “collisionless” constants of the motion — \( p_\zeta \propto \psi \quad \Rightarrow \quad f_0 = f(\psi) \)

- BUT WITH RESISTIVITY \( \vec{B}_{\text{poloidal}} = \vec{\nabla}\zeta \times \vec{\nabla} \psi \) is a dissipative (i.e., non-conservative) electromagnetic field: can’t integrate over \( t \), \( p_\zeta \) not conserved
Key Paleoclassical Hypothesis (continued)

- Mathematically, the poloidal flux $\psi$ obeys a diffusion equation (for $|x| > \delta_c \equiv c/\omega_p \sim 0.1$ mm — outside $em$ skin depth) on the resistive time scale:

$$\frac{d\psi}{dt} = D_\eta \Delta^+ \psi - \frac{\partial \Psi}{\partial t} \approx D_\eta \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} - \frac{\partial \Psi}{\partial t}, \quad D_\eta \equiv \frac{\eta_{\text{nc}}}{\mu_0}, \quad \text{magnetic field diffusivity}$$

- On electron gyromotion time scale $p_\xi = m_e R^2 \dot{\xi} - q_e \psi$ is conserved

$$\Rightarrow \quad \psi(x, t) = \psi(x_g) + \delta \psi \sin(-\omega_{ce} t + \varphi_0), \quad \delta \psi \sim q_e d\psi/d\rho \ll \psi(x_g)$$

- But small poloidal magnetic flux $\delta \psi$ traversed by radial component of electron gyromotion is governed by a diffusion equation and diffuses radially:

$$\frac{d\delta \psi}{dt} = D_\eta \frac{\partial^2 \delta \psi}{\partial x^2} \Rightarrow \quad \delta \psi(x, t) = \frac{\exp[-(x - x_0)^2/4D_\eta t]}{(4\pi D_\eta t)^{1/2}} \Rightarrow \quad \frac{\langle (\Delta x_\psi)^2 \rangle}{\Delta t} = 2D_\eta$$

- In an electron gyroperiod $\tau \equiv 2\pi/\omega_{ce}$ this causes the electron guiding center to diffuse by $\langle (\Delta x_g)^2 \rangle = 2D_\eta \tau$, which causes $(\tau = \Delta t)$

$$\frac{\langle (\Delta x_g)^2 \rangle}{\Delta t} = 2D_\eta, \quad \text{Fokker-Planck coefficient for electron guiding center diffusion}$$
Paleoclassical Model Adds Guiding Center Diffusion Effects To Drift-Kinetic Equation Via A Fokker-Planck Operator

- Magnetic-field-diffusion Modified Drift-Kinetic Equation (MDKE) adds Fokker-Planck diffusion operator to drift-kinetic equation for electrons:

\[
\frac{\partial f}{\partial t} + \frac{v_B}{B} \vec{\nabla} f + \vec{v}_D \cdot \vec{\nabla} f + \hat{\epsilon} \frac{\partial f}{\partial \epsilon} = C\{f\} + D\{f\}
\]

- Neglecting advective effects due to the “grid velocity” and \(d\psi/dt \neq 0\), the Fokker-Planck (F-P) diffusion coefficient is given by:

\[
D\{f\} \equiv \vec{\nabla} \cdot \left( \frac{\vec{\nabla} \cdot \left( \Delta \vec{x} \Delta \vec{v} \right)}{2\Delta t} f \right) \simeq \frac{1}{V'} \frac{\partial^2}{\partial \rho^2} (V'D_\eta f), \quad \text{Fokker-Planck diffusion operator}
\]

- Paleoclassical electron heat flux results from flux-surface-average of energy moment of this F-P diffusion operator for a lowest order Maxwellian:

\[
\langle \vec{\nabla} \cdot \vec{q}_e^{pc} \rangle \simeq \left\langle \int d^3v \frac{m_e v^2}{2} D\{ \int_{-L}^{L} \frac{dl}{2\pi R_0 q} f_{\text{Max}} \} \right\rangle \simeq \frac{1}{V'} \frac{\partial^2}{\partial \rho^2} \left[ V' \chi_e^{pc} / a^2 \right] n_e T_e
\]

\[
\chi_e^{pc} \simeq \frac{3}{2} \frac{L}{\pi R_0 q} D_\eta \quad \text{in which } L \text{ is half-length over which Maxwellian is equilibrated}
\]
Motivation For Paleoclassical Studies Near Separatrix

- Since \( D_\eta \propto \eta \propto 1/T_e^{3/2} \), \( \chi_{e}^{pc} \) in the confinement region is typically
  \[
  \chi_{e}^{pc} \sim 1.5 \frac{Z_{\text{eff}}}{[T_e(\text{keV})]^{3/2}} \frac{m^2}{s} \gtrsim 1 \text{ m}^2/\text{s for } T_e \lesssim 2 \text{ keV}
  \]

- Microturbulence-induced transport usually has a gyroBohm scaling:
  \[
  \text{ITG, DTE: } \chi_{i}^{gB} \equiv \# \frac{q_i}{a} \frac{T_e}{eB} \simeq 3 \# \frac{[T_e(\text{keV})]^{3/2} A_i^{1/2}}{a(m) [B(T)]^2} \frac{m^2}{s} \gtrsim 1 \text{ m}^2/\text{s for } T_e \gtrsim 0.5 \text{ keV}/\#^{2/3}
  \]
  \[
  \text{ETG: } \chi_{e}^{gB} \equiv \# \frac{q_e}{a} \frac{T_e}{eB} \simeq 0.1 \#_e \frac{[T_e(\text{keV})]^{3/2}}{a(m) [B(T)]^2} \frac{m^2}{s} \gtrsim 1 \text{ m}^2/\text{s for } T_e \gtrsim 5 \text{ keV}/\#^{2/3}_e
  \]

- Thus, paleoclassical \( \chi_{e}^{pc} \) is likely to be dominant for \( T_e \lesssim 1 \text{ keV} \) \((B^{2/3}/\#^{1/3})\)

- In DIII-D the electron temperature \( T_e \) in the edge pedestal region ranges from about 100 eV at the separatrix to about 1 keV at top of pedestal

\[\implies\] paleoclassical \( \chi_{e}^{pc} \) likely to be dominant in pedestal region
**Paleoclassical Electron Heat Diffusivity Near A Separatrix**

- The length $L$ in $\chi_e^{pc}$ is the minimum of the following half-lengths:

  \[
  \ell_{\text{max}} \simeq \pi R_0 q n_{\text{max}}, \text{ with } n_{\text{max}} \simeq (\pi \delta_e q')^{-1/2} \quad \text{— maximum diffusing field line length}
  \]
  \[
  \ell_{n^o} \simeq \pi R_0 q n^o, \text{ for } n^o = 1, 2 \quad \text{— line length on low order rational surface (} q_* \equiv m^o / n^o \text{)}
  \]
  \[
  \lambda_e \equiv v_{Te}/\nu_e \quad \text{— electron collision length}
  \]
  \[
  \pi R_0 q \quad \text{— poloidal periodicity length}
  \]

- Near a magnetic separatrix in a divertor plasma many things happen:

  \[
  q \text{ and } q' \text{ get very large} \quad \Rightarrow \quad \text{small } n_{\text{max}} \lesssim 1 \quad \Rightarrow \quad L \simeq \lambda_e \text{ but } \lambda_e \lesssim \pi R_0 q
  \]

- Taking account of the additional contribution due to the toroidal periodicity there (for $\lambda_e > \pi R$), the net paleoclassical electron heat diffusivity inside but near a magnetic separatrix is (for $\pi R < \lambda_e < \pi R_0 q \max\{1, n_{\text{max}}\}$)

  \[
  \chi_e^{pc} \simeq \frac{3}{2} \frac{\eta_{||}^{Sp}}{\mu_0} \left( 1 + \frac{\eta_{||}^{nc}}{\eta_{||}^{Sp} \pi R_0 q} \right) \frac{\lambda_e}{\eta_{||}^{Sp} \pi R_0 q}, \text{ in which } \eta_{||}^{Sp}, \eta_{||}^{nc} \text{ are the Spitzer, neoclassical resistivity}
  \]

*JDC/PaleoGA — 2/9/05, p 6*
Approximate Forms Of $\chi_{e}^{pc}$ Predictions Near A Separatrix

• On the separatrix $\chi_{e}^{pc}$ is just the toroidal periodicity result:

$$
\chi_{e}^{pc} \approx \frac{3}{2} \frac{\eta_{||}^{Sp}}{\mu_0} \approx Z_{eff} \left[ \frac{100}{T_e(eV)} \right]^{3/2} \frac{m^2}{s}
$$

$$
\implies \quad 2 \text{ m}^2/\text{s} \text{ for DIII-D separatrix assuming } T_{e_{sep}} \approx 100 \text{ eV and } Z_{eff} \approx 2
$$

• Moving inside the separatrix $\chi_{e}^{pc} \propto T_e^{-3/2}$ decreases as $T_e$ increases until $q$ decreases to less than $q(\rho^s) \equiv (\lambda_e/\pi R_0)(\eta_{||}^{nc}/\eta_{||}^{Sp}) \sim 3$–$30$ in DIII-D

$$
\implies \quad \text{for DIII-D } \chi_{e}^{pc} \text{ decreases as } T_e^{-3/2} \text{ until } q \lesssim 3 \text{ (L-mode) or 30? (H-mode)}
$$

• Further into the plasma $\chi_{e}^{pc}$ is in the “collisional paleoclassical regime” and has Alcator-type scaling:

$$
\chi_{e}^{pc} \approx \frac{3}{2} \frac{\eta_{||}^{nc}}{\eta_0} \frac{v_{Te}}{\pi R_0 q} \frac{e^2}{\omega_p^2} \propto \frac{T_e^{1/2}}{n_e q}, \text{ until } \lambda_e > \pi R_0 q n_{max} \sim 20R_0 q \sim 100 \text{ m at } q \sim 3,
$$

beyond which $\chi_{e}^{pc} \propto T_e^{-3/2}$ and turbulence-induced transport likely becomes important
Simplified Transport Analysis Can Be Used Near Separatrix

• Assuming the dominant loss is due to the radial electron heat flux $\bar{q}_e$, the equilibrium electron energy balance equation is simply

$$\frac{\partial}{\partial V} \langle \bar{q}_e \cdot \nabla V \rangle = Q_e, \text{ net electron heating (collisional, joule + auxiliary, minus radiation)}$$

• Integrating this over volume yields $\langle \bar{q}_e \cdot \nabla V \rangle \rho = \int_{V(\rho)} d^3x Q_e \equiv P_e(\rho)$

• Assuming the paleoclassical electron heat flux dominates, this result can be integrated inward from the separatrix at $\rho = 1$ to obtain $T_e(\rho)$:

$$-(\partial/\partial \rho)[V'(\chi_e^{pc}/\bar{a}^2)n_e T_e] = P_e(\rho) \implies -V'(\chi_e^{pc}/\bar{a}^2)n_e T_e \big|_{\rho}^1 = \int_{\rho}^1 d\rho P_e(\rho) \simeq (1-\rho)P_e(1)$$

• Very near the separatrix where $\chi_e^{pc} \sim T_e^{-3/2}$ this result yields

$$T_e(\rho) = \left[ \frac{n_e(\rho) [V'/\bar{a}^2]_\rho 10^3 Z_{\text{eff}}}{n_e(1) [V'/\bar{a}^2]_{\rho=1} 10^3 Z_{\text{eff}} T_e(1)^{-1/2} + (1-\rho) P_e(1)/e} \right]^2 \simeq T_e(1) \left[ \frac{n_e(\rho)}{n_e(1)} \right]^2$$

$$\implies \text{ in DIII-D the paleoclassical } \chi_e^{pc} \text{ model predicts } T_e(\rho) \propto n_e^2(\rho) \text{ from separatrix inward until } q \lesssim q(\rho^*) = 3 \text{ (L-mode) or 30 (H-mode)}$$
Comparison To DIII-D Near-Separatrix Data (Next 2 VGs)

- Paleoclassical predictions to be compared with DIII-D data are:

  1) positive curvature ($\partial^2 T_e/\partial \rho^2 > 0$) of $T_e$ profile near separatrix — since $\chi_e^{pc} \sim 1/T_e^{3/2}$,
  2) gradient of $T_e$ increases moving inward from separatrix — until $q(\rho) < q(\rho^s) \sim 3$–30,
  3) At $\rho = \rho^s \sim 0.95$–0.98, $T_e(\rho^s) \simeq [n_e(\rho^s)/n_e(1)]^2 T_e(1) \implies \eta_e \equiv L_{n_e}/L_{T_e} = 2$,
  4) collisional regime (Alcator-type scaling) for $\rho \leq \rho^s \implies T_e$ independent of $n_e$.

- DIII-D L-mode data (D.G. Whyte et al. submitted to PPCF 10/04):

  $T_e(1) \simeq 100$ eV, max $\vec{\nabla} T_e$ at 2 cm inside separatrix, $[n_e(\rho^s)/n_e(1)]^2 \simeq 1.2^2 = 1.44$

  $\implies$ 1) $\partial^2 T_e/\partial \rho^2 \gtrsim 0$, 2) max $\vec{\nabla} T_e$ at $\rho^s \simeq 0.95$, 3) $T_e(\rho^s) \simeq 150$ eV, 4) $T_e \propto n_e$

- DIII-D H-mode data (T.H. Osborne, private communications 2/04, 2/9/05):

  $T_e(1) \simeq 100$ eV, max $\vec{\nabla} T_e$ about 0.8 cm inside separatrix, $[n_e(\rho^s)/n_e(1)]^2 \simeq 6$

  $\implies$ 1) $\partial^2 T_e/\partial \rho^2 > 0$, 2) max $\vec{\nabla} T_e$ at $\rho^s \simeq 0.98$, 3) $T_e(\rho^s) \simeq 500$ eV, 4) ? on $T_e \propto n_e$

- Paleoclassical predictions for $T_e$ pedestal profile shape, width:

  1) For $\rho \geq \rho^s$, $T_e(\rho) \sim n_e(\rho)^2 \sim \tanh^2(\delta x/\Delta_n)$, 2) $1 - \rho^s$ is only relevant width parameter

† DIII-D L-mode data (D.G. Whyte et al. submitted to PPCF 10/04):

$T_e(1) \simeq 100$ eV, max $\vec{\nabla} T_e$ at 2 cm inside separatrix, $[n_e(\rho^s)/n_e(1)]^2 \simeq 1.2^2 = 1.44$

$\implies$ 1) $\partial^2 T_e/\partial \rho^2 \gtrsim 0$, 2) max $\vec{\nabla} T_e$ at $\rho^s \simeq 0.95$, 3) $T_e(\rho^s) \simeq 150$ eV, 4) $T_e \propto n_e$

† DIII-D H-mode data (T.H. Osborne, private communications 2/04, 2/9/05):

$T_e(1) \simeq 100$ eV, max $\vec{\nabla} T_e$ about 0.8 cm inside separatrix, $[n_e(\rho^s)/n_e(1)]^2 \simeq 6$

$\implies$ 1) $\partial^2 T_e/\partial \rho^2 > 0$, 2) max $\vec{\nabla} T_e$ at $\rho^s \simeq 0.98$, 3) $T_e(\rho^s) \simeq 500$ eV, 4) ? on $T_e \propto n_e$

† Paleoclassical predictions for $T_e$ pedestal profile shape, width:

  1) For $\rho \geq \rho^s$, $T_e(\rho) \sim n_e(\rho)^2 \sim \tanh^2(\delta x/\Delta_n)$, 2) $1 - \rho^s$ is only relevant width parameter
1) For $-20 \text{ mm} \leq x \leq 0$, 
$\partial^2 T_e / \partial \rho^2 \gtrless 0$, 
positive or neutral curvature

2) $\vec{\nabla} T_e$ changes around 
$x \simeq -20 \text{ mm} \ (\rho^s \sim 0.95)$

3) $T_e(-20\text{mm}) \sim 150 \text{ eV}$ 
$\sim 1.5 \ T_{\text{sep}}$, while 
$[n_e(\rho^s)/n_e(1)]^2 \sim 1.2^2 = 1.44$

4) For $x < -20 \text{ mm}$ 
$T_e$ is independent of $n_e$

Top of Fig. 4 in D.G. Whyte et al., “The magnitude of plasma flux to the main wall in the DIII-D tokamak,” 
October 2004 (to be published in PPCF)
1) For $2.274 \lesssim R \lesssim 2.282$ ($\sim R_{\text{sep}}$), $\partial^2 T_e/\partial \rho^2 \gtrsim 0$, positive curvature

2) $\nabla T_e$ maximum around $R \approx 2.274$ ($\sim 0.08 \text{ cm}$ inside separatrix, $\rho^s \gtrsim 0.98$)

3) $T_e(2.274) \sim 500 \text{ eV}$
   $\sim 5 T_{e\text{sep}}$, while
   $[n_e(\rho^s)/n_e(1)]^2 \sim 6$

4) For $R < 2.274$ on $T_e$ dependence on $n_e$

Data courtesy of T.H. Osborne (unpublished, 2004, and most importantly on 2/9/05)
Possible Ways To Increase Pedestal $T_e$ Height And Width

- To increase pedestal $T_e$ height, since paleoclassical prediction is $T_e(\rho) \simeq [n_e(\rho)/n_e(1)]^2 [(Z_{\text{eff}} V'/\bar{a}^2)_{\rho}/(Z_{\text{eff}} V'/\bar{a}^2)_{1}]^2$:
  1) Reduce electron density on separatrix, i.e., $n_e(1)$
  2) Increase $n_e$ at $\rho^s \approx 0.98$ [where $q(\rho^s) \sim 30?$]
  3) Increase ratio of $Z_{\text{eff}} V'/\bar{a}^2$ at $\rho^s \approx 0.98$ relative to its separatrix value

- To increase $T_e$ pedestal width, since position of maximum $\nabla T_e$ is determined from $q(\rho^s) [\pi R_0 (\eta^\text{Sp}/\eta^\text{nc})] \gtrsim \lambda_e \propto T_e^2/n_e$:
  1) Keep $q$ as large as possible as one moves inward from the separatrix
     $\implies$ keep $|\nabla \psi|$ very small near separatrix, perhaps via unbalanced double null divertor with split separatrices, or via local ECCD?
  2) Maybe use weak ergodic magnetic divertor to produce $q \rightarrow \infty$ in high $n_e$ region
     — as long as its magnetic flutter does not increase $\chi_e$
SUMMARY

- Paleoclassical radial electron heat transport is likely to be dominant for $T_e \lesssim 1$ keV — hence in DIII-D edge pedestal region.

- Near the separatrix the paleoclassical electron heat diffusivity is

$$\chi_{e}^{pc} \approx \frac{3}{2} \frac{\eta_{Sp}^{nc}}{\mu_0} \left(1 + \frac{\lambda_e}{\pi R_0 q} \frac{\eta_{Sp}^{nc}}{\eta_{Sp}^{nc}}\right) \approx Z_{eff} \left[\frac{100}{T_e(eV)}\right]^{3/2} \left(1 + \frac{10^{16}[T_e(eV)]^2}{Z_{eff}}\right) \frac{m^2}{s}$$

- Near the separatrix the paleoclassical model predicts:
  1) $\chi_{e}^{pc} \propto 1/T_e^{3/2}$ (decreases moving to higher $T_e$) and $\partial^2 T_e/\partial \rho^2 > 0$ (positive curvature),
  2) maximum $\vec{\nabla} T_e$ where $q(\rho^s) \sim (\lambda_e/\pi R_0)(\eta_{nc}^{nc}/\eta_{Sp}^{nc}) \sim 3-30$,
  3) $T_e(\rho^s) \sim T_e(1) \left[n_e(\rho^s)/n_e(1)\right]^2 \propto n_e^2(\rho^s)$, and
  4) For $\rho \leq \rho^s$, $\chi_{e}^{pc} \sim T_e^{1/2}/n_e q$ (Alcator scaling).

- Preliminary comparisons with DIII-D L-mode and H-mode pedestal data are encouraging.
Annotated Bibliography

• Basic paleoclassical model in a sheared-slab magnetic field geometry:

• Axisymmetric magnetic field geometry, full kinetic analysis:

• Paleoclassical model, plus experimental comparisons and possible tests:
  “Paleoclassical electron heat transport” (submitted to Nuclear Fusion); UW-CPTC 04-9, December 13, 2004 (expanded version of 2004 IAEA Vilamoura paper TH/1-1, UW-CPTC 04-8, October 2004).

• Preprints of these papers are available from
  http://homepages.cae.wisc.edu/~callen
  http://www.cptc.wisc.edu