Experimental Tests Of Paleoclassical Transport


1University of Wisconsin, Madison, WI 53706-1609 USA
2Cal. Poly. St. U., San Luis Obispo, CA 93407 USA
3General Atomics, San Diego, CA 92186-5608 USA
4Lehigh University, Bethlehem, PA 18015-3182 USA
5Princeton Pl. Ph. Lab., Princeton, NJ 08543-0451 USA
6JAEA, Naka, Ibaraki-ken, 311-0193, JAPAN
7Mass. Inst. of Tech., Cambridge, MA 02139 USA
8FOM Inst. for Pl. Phys. Rijnhuizen, NETHERLANDS
9Oak Ridge Nat. Lab., Oak Ridge, TN 37831 USA
10Lawr. Liv. Nat. Lab., Livermore, CA 94551-0808 USA
11Georgia Tech, Atlanta, GA 30332 USA
12Johns Hopkins Univ., Baltimore, MD 21218 USA

Theses:

• Tests: The paleoclassical model for radial electron heat transport in resistive, current-carrying toroidal plasmas has been tested mainly against well-characterized, previously published experimental data.

• Regimes: Paleoclassical transport can dominate at low $T_e$ where it exceeds gyro-Bohm-level anomalous transport: $T_e \lesssim T_e^{\text{crit}} \approx B(T)^{2/3} \bar{a}(m)^{1/2}$ keV.

• Conclusions: Paleoclassical radial electron heat transport may set lower limit on electron heat transport (within factor $\sim 2$) in many toroidal plasmas — in ohmic-level plasmas, eITBs, H-mode edge pedestals, and when micro-turbulence-induced transport is small.

*aThis poster and the corresponding 2006 IAEA Chengdu paper EX/P3-2 are available from http://homepages.cae.wisc.edu/~callen.
Model: **Paleoclassical Radial Electron Heat Transport**

- Paleoclassical radial electron heat transport operator $\langle \vec{\nabla} \cdot \vec{Q}^{pc}_{e} \rangle$ is
  \[
  \frac{\partial}{\partial V} \langle \vec{Q}^{pc}_{e} \cdot \vec{\nabla} V \rangle = - \frac{M + 1}{V'} \frac{\partial^2}{\partial \rho^2} \left( V' \bar{D}_\eta \frac{3}{2} n_e T_e \right), \quad M \equiv \frac{L}{\pi \bar{R} q} = \min \{ \ell_{\text{max}}, \lambda_e, \ell_n^{\circ} \} \lesssim 10.
  \]

- The diffusive part indicates a paleoclassical electron heat diffusivity of
  \[
  \chi^{pc}_{e} \equiv \frac{3}{2} (M + 1) D_\eta \quad \Rightarrow \quad \begin{cases} 
  \frac{3}{2} \left( \frac{1}{\pi \delta_e |q'|} \right)^{1/2} \frac{\eta_{||}^{nc}}{\mu_0}, & \text{collisionless (} M \sim 10) \\
  \frac{3}{2} \frac{\eta_{||}^{nc}}{\eta_0} \frac{v T_e}{\pi \bar{R} q} \frac{c^2}{\omega_p^2}, & \text{collisional (} 10 \gtrsim M > 1) \\
  \frac{10^3 Z_{\text{eff}}}{T_e (\text{eV})^{3/2}}, & \text{near separatrix (} M < 1)
  \end{cases}
  \]

$D_\eta \equiv \eta_{||}^{nc} / \mu_0$ is magnetic diffusivity, $\eta_{||}^{nc}$ is parallel neoclassical resistivity, $\lambda_e$ is collision length and $\ell_{\text{max}} = \pi \bar{R} q n_{\text{max}}$ is diffusing field line length with $n_{\text{max}} \equiv (\pi \delta_e |q'|)^{-1/2}$.

- $\chi^{pc}_{e}$ is small in vicinity of low-order rational surfaces with $q^\circ \equiv m^\circ / n^\circ$:
  \[
  \chi^{pc}_{e} \sim \frac{3}{2} (n^\circ + 1) \frac{\eta_{||}^{nc}}{\mu_0}, \quad \text{over width} \quad 2 \delta x_{\text{min}}^\circ \sim 2 \left( \frac{2}{n^\circ} \right)^{2/3} \left( \frac{\pi \delta_e}{q''} \right)^{1/3} \text{around} \ q_{\text{min}}.
  \]
**Regime: Paleoclassical Transport Likely Dominates At Low $T_e$**

- Since $D_\eta \propto \eta \propto 1/T_e^{3/2}$, $\chi_{e{\text{pc}}}$ in the confinement region is typically
  
  $$\chi_{e{\text{pc}}} \sim \frac{Z_{\text{eff}}[\tilde{a}(m)]^{1/2}}{[T_e(\text{keV})]^{3/2}} \frac{m^2}{s} \gtrsim 1 \text{ m}^2/\text{s for } T_e \lesssim 2 \text{ keV}.$$  

- Microturbulence-induced transport usually has a gyroBohm scaling:
  
  $$\chi_{e{\text{gb}}} \equiv f_\# \frac{\varrho_s T_e}{a eB} \sim 3.2 f_\# \frac{[T_e(\text{keV})]^{3/2} A_i^{1/2}}{\tilde{a}(m) [B(T)]^2} \frac{m^2}{s} \gtrsim 1 \text{ m}^2/\text{s for } T_e \gtrsim 0.5 \text{ keV}/f_\#^{2/3}.$$  

- Thus, paleoclassical $\chi_{e{\text{pc}}}$ is likely to be dominant for low $T_e$:
  
  $$T_e \lesssim \boxed{T_{e{\text{crit}}} \equiv [B(T)]^{2/3} [\tilde{a}(m)]^{1/2}/(3 f_\#)^{1/3} \text{ keV}} \sim 0.7-2.4 \text{ keV (} f_\# \sim 1/3\text{), present expt.}$$

- In DIII-D the electron temperature $T_e$ in the edge pedestal region ranges from about 100 eV at the separatrix to about 1 keV at top of pedestal
  
  $$\implies \text{paleoclassical } \chi_{e{\text{pc}}} \text{ likely to be dominant in H-mode pedestal region.}$$

- In ITER $T_{e{\text{crit}}} \sim 5 \text{ keV}$  
  
  $$\implies \text{paleoclassical may be dominant for ITER ohmic startup and in pedestal region?}$$
Core, Edge: **Paleoclassical $\chi_e$ Agrees With C-Mod H-Mode $\chi_{\text{eff}}$**

- **Sawteeth influence**
  \[ \rho < \rho_{\text{inv}} \simeq 0.35 \]

- **I**: Collisionless paleo for
  \[ \rho < 0.43, \text{ where } M \sim 15 \]

- **II**: Collisional paleo for
  \[ 0.45 < \rho < 0.85, \text{ where } 15 > M > 1 \text{ and } T_e < T_e^{\text{crit}} \simeq 1.6 \text{ keV} \]

- **III**: Edge paleo for
  \[ \rho > 0.85, \text{ where } M < 1 \]

- **Paleoclassical $\chi_e$ agrees reasonably well in all 3 regimes**

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**Figure 1**: Comparison of $\chi_{\text{pc}}^e$ with TRANSP-analyzed C-Mod data from H-mode shot 960116027, Greenwald et al., NF **37**, 793 (1997).
Core: $\chi_e^{pc}$ Agrees With DIII-D Ohmic-Level Beta-Scan $\chi_e$

- Sawteeth influence
  $\rho < \rho_{inv} \approx 0.4$ (shaded)

- Collisionless paleo for
  $\rho < 0.8$, where $M \sim 8$

- Collisional paleo for
  $\rho > 0.8$, where $8 > M \gtrsim 1$

- Dynamic modeling with $\chi_e^{pc}$
  for $0.4 < \rho < 0.9$ yields experimental $T_e(\rho) \pm 10\%$

- Other ohmic-level cases,
  $T_e(0.4) \lesssim T_e^{crit} \lesssim 1.35$ keV, in $\beta$, $\nu_*$ scans also agree $\pm \times 2$

- But in hybrid shots with
  $T_e(0.5) \sim 2.5$ keV $\gg T_e^{crit}$,
  $\chi_e^{pc} \lesssim \chi_e^{exp}/5 \ll \chi_e^{exp}$

Figure 2: Comparison of $\chi_e^{pc}$ with ohmic-level DIII-D discharge 90118 from beta-scan, Petty et al., Nucl. Fusion 38, 1183 (1998).
Core: $\chi_{e}^{pc}$ Consistent With DIII-D LOC But Not SOC Plasmas

- In Linear Ohmic Confinement (LOC) plasmas $\tau_E \sim n_e$, and DIII-D plasmas are partially in collisional paleoclassical regime and in reasonable agreement for the key confinement region $0.5 \lesssim \rho \lesssim 0.8$ — see Fig. 3 below.

- In higher density Saturated Ohmic Confinement (SOC) DIII-D plasmas with ITG modes, $\chi_{e}^{pc}$ is smaller with wrong profile in this region — Fig. 4.

Figure 3: DIII-D electron heat diffusivity in LOC regime: analysis (green), paleo (red), $2 \times$ paleo (blue). Discharge from Rettig et al., Phys. Plasmas 8, 2232 (2001).

Figure 4: DIII-D electron heat diffusivity in SOC regime: analysis (green), paleo (red), $2 \times$ paleo (blue). Discharge from Rettig et al., Phys. Plasmas 8, 2232 (2001).
**Between Sawteeth: In DIII-D Bean-shaped Plasmas**

\[ \chi_{e}^{\text{exp}} \downarrow \chi_{e}^{\text{pc}} \]

- In DIII-D bean-shaped sawtoothing plasmas, just before sawtooth crash at 3.03 s, \( \chi_{e} \sim \chi_{e}^{\text{pc}} \) despite \( T_{e}(0) \simeq 2.5 \text{ keV} \gg T_{e}^{\text{crit}} \simeq 1.3 \text{ keV} \) — Fig. 5
- Temporally, the \( \chi_{e} \) averaged over the hot core region \( (\rho \leq 0.33) \) decays to the paleoclassical prediction just before the next sawtooth crash — Fig. 6

![Figure 5](image1.png)

*Figure 5: TRANSP analysis of DIII-D \( \chi_{e} \) for bean cross-section plasma at 3.03 s, just before a sawtooth crash. Discharge from Lazarus et al., PPCF 48, L65 (2006).*

![Figure 6](image2.png)

*Figure 6: Temporal decay of DIII-D core-average \( \chi_{e} \) for bean cross-section plasma to next sawtooth crash. Dis-charge from Lazarus et al., PPCF 48, L65 (2006).*
eITBs: RTP eITBs At Low Order Rationals Modeled by $\chi_{e}^{pc}$

- Near low order rational surfaces ($q^o \equiv m^o/n^o$, $n^o = 1, 2$), $\chi_{e}^{pc}$ is smaller (p2) and produces electron Internal Transport Barriers (eITBs) — Fig. 7
- “Stair-step” $T_e(0)$ in RTP experiments as highly localized ECH deposition was moved outward is modeled well with $2 \times \chi_{e}^{pc} - T_e^{crit} \simeq 0.7$ keV, Fig. 8

Figure 7: Profiles for RTP ohmic discharge: intially (blue, experimental $T_e$), paloclassical modeling after 50 ms (red). Largest eITBs are at $q = 1/1, 2/1, 3/1$.

Figure 8: $T_e$ on axis as ECH deposition is moved radially outward: RTP exp. [blue, PPCF 39, B303 (1997)], paleo modeling (red with sawtooth model, orange without).
eITBs: RTP eITBs Take Magnetic Diffusion Time To Develop

- Paleoclassical model is very sensitive to local $q$ profile — Fig. 9; hence it should take a magnetic “skin” diffusion time to come into equilibrium
- Temporal development of central $q$ and $T_e$ are well modeled by $2 \times \chi_{e}^{pc}$ and take $\tau_{\eta} \equiv a^2/6D_{\eta}(\rho = 0) \sim 20$ ms to come to equilibrium — Fig. 10

![Figure 9: Equilibrium paleoclassical modeling profiles of $T_e$, $q$ and $\chi_e$ in RTP for ECH deposition at $\rho_{dep} = 0.446$ (red) and 0.447 (blue); Hogeweij et al., 2006 EPS.](image1)

![Figure 10: Evolution of central $T_e$, $q$ for ECH $\rho_{dep} = 0.446$ (red) and 0.447 (blue): RTP experiment (solid) and paleo modeling (dashed); Hogeweij et al., 2006 EPS.](image2)
**eITB**: Strong eITB in JT-60U May Be Caused By Strongly Reversed Magnetic Shear In Core

- Strong reversed shear inside $q_{\text{min}}$ decreases collisionless $\chi_{e}^{pc} \sim |q'|^{-1/2}T_{e}^{-3/2}$
- When micro-turbulence-induced transport is low, $\chi_{e}^{pc}$ can provide a low minimum value of electron heat diffusivity inside $q_{\text{min}}$ — Figs. 11, 12, 18

![Figure 11: $T_{e}$, $q$ TRANSP profiles in JT-60U for strong eITB inside of $q_{\text{min}}$ at $\rho \simeq 0.575$; $T_{e}^{\text{crit}} \simeq 2.4$ keV.](image)

![Figure 12: TRANSP and paleo $\chi_{e}$ for the JT-60U case in Fig. 11, like those in Fujita et al., PRL 78, 2377 (1997).](image)
Pedestal: **H-Mode Edge Pedestals In DIII-D Involve Collisional And Near Separatrix Regimes of $\chi_{pc}^e$**

- At top of pedestal ($\rho \approx 0.95$)
  \[ \lambda_e \approx 80 \text{ m} < \ell_{\text{max}} \approx 160 \text{ m} \]
  \[ \implies \text{collisional pc regime} \]

- At $T_e$ symmetry ($\rho \approx 0.978$)
  \[ \lambda_e \approx 40 \text{ m} \sim \pi R_0 q \approx 22 \text{ m} \]
  \[ \implies \sim \text{transition} \]

- Outside $T_e$ symmetry point
  \[ \pi R_0 q > \lambda_e > \pi R_0 \approx 5 \text{ m} \]
  \[ \implies \text{near separatrix pc regime} \]

- Outside $T_e$ symmetry point
  the decreasing $T_e$ increases $\chi_{pc}^e \sim T_e^{-3/2}$, which causes
  \[ \frac{\partial^2 T_e}{\partial \rho^2} \geq 0 \ (\text{+ curvature}) \]

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**Figure 13**: Edge pedestal $n_e$ and $T_e$ profiles for DIII-D shot 98889, averaged over 80-99% of time to next ELM crash, at $\sim 4500$ ms. Dots on curves indicate symmetry points for $\tanh$ fits.
**Pedestal:** Paleoclassical Model Is In Reasonable Agreement With Transport Analysis $\chi_e$ In DIII-D H-Mode Pedestals

- Transport analysis $\chi_e$ depends on electron fraction of the heat flow through the separatrix: $(Q_e/Q)_{\text{sep}}$

- Collisional paleo regime for $\rho < 0.94$, where $5 > M > 0.5$

- Near separatrix paleo regime for $\rho > 0.94$, where $M < 0.5$

- Paleoclassical $\chi_e$ agrees well in edge, and in collisional regime for $(Q_e/Q)_{\text{sep}} = 0.4$

![Figure 14: Transport analysis $\chi_e$ for a DIII-D H-Mode pedestal compared to paleoclassical. DIII-D data from W.M. Stacey and R.J. Groebner, Phys. Plasmas 13, 072510 (2006).](image-url)
Pedestal: *ASTRA Modeling of DIII-D Shot 98889 Illustrates Role Of Micro-Turbulence Versus Paleoclassical In Edge*

- ITG/TEM $\chi_e$ dominant for $\rho < 0.9$ — in core
- $\vec{E} \times \vec{B}$ flow shear has been used to reduce edge turb. transport
- Paleoclassical $\chi_e$ dominant for $\rho > 0.9$ — edge
- $T_e(\rho)$ modeled well by combo of ITG/TEM (core) and paleo (edge)
- Increasing $\chi_e^{pc}$ as one approaches separatrix causes $\sim$ neutral curvature of $T_e$ for $\rho > 0.975$

Figure 15: ASTRA modeling [Pankin et al., NF 46, 403 (2006)] of DIII-D edge $T_e(\rho)$ in shot like the 98889 discharge shown in Fig. 13. Last grid point at $\rho = 0.997$ is set to an artificially low boundary $\chi_e$ value.
Pedestal: Paleo Model Predicts $T_e \propto n_e^2$, $\beta_e^{\text{ped}}$ Scaling In DIII-D

- Near the separatrix $M << 1$ and $n_e T_e D \eta \propto n_e / T_e^{1/2}$, which yields $T_e \propto n_e^2$ and $\eta_e \equiv \partial \ln T_e / \partial \ln n_e = 2$ paleoclassical predictions there — Fig. 16

- Pedestal $T_e$ predicted by balancing collisional $\chi_e^{\text{pc}} (p2)$ against $\chi_e^{gB} (p3)$ is $\beta_e^{\text{ped}} \equiv n_e^{\text{ped}} T_e^{\text{ped}} / (B^2 / 2 \mu_0) \simeq (0.032 / f_\# A_i^{1/2}) \left( \bar{a} / R_q \right) (\eta_{nc} / \eta_0)$ — Fig. 17

Figure 16: DIII-D database of $T_e$ vs. $n_e^2$ at symmetry point is consistent with paleoclassical prediction for $T_e \lesssim 0.2$ keV. Also, prediction bounds data at all $T_e$.

Figure 17: DIII-D database of $\beta_e^{\text{ped}}$ is reasonably consistent with paleoclassical model prediction for $f_\# \simeq 0.8$. Range of $f_\#$ that bounds the data is $0.5 < f_\# < 2$. 

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Spherical Tokamaks: NSTX L-Mode Comparisons With Paleo

- Decease of $\chi_e$ in reversed shear region ($\rho < 0.45$) of an ohmic-level NSTX L-mode plasma is captured by paleoclassical model — Fig. 18 (& 11, 12)
- Minimum $\chi_e$ at $\rho = 0.65$ is set by $\chi_e^{pc}$ for $T_e \lesssim T_e^{crit} \sim 0.6 B^{2/3}$ keV; micro-turbulence-induced transport likely dominates at higher $T_e$ — Fig. 19

**Figure 18:** Comparison of TRANSP $\chi_e$ is similar to paleoclassical prediction for an ohmic-level NSTX L-mode plasma [Stutman et al. (to be published)]. Dotted line indicates special consideration near minimum in $q$.

**Figure 19:** Database of ratio of TRANSP to paleoclassical $\chi_e$ for 2004 and 2005 NSTX campaigns. It is near unity below $T_e \sim 600 B_T^{2/3}$ eV but $> 1$ above it, which implies paleo is minimum for $T_e \lesssim T_e^{crit}$ with $f_# \sim 1$. 

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RFPs, Spheromaks: MST and SSPX Comparisons With $\chi_e^{pc}$

- In magnetically quiescent PPCD reversed field pinch plasmas in MST, $\chi_e$ is not lower than $\chi_e^{pc}$ and has approximately the same profile — Fig. 20
- SSPX spheromak transport at low $T_e$ is dominated by magnetic fluctuations and Rechester-Rosenbluth level transport, but for $T_e \gtrsim 200$ eV fluctuations decrease and paleoclassical may set lower limit on $\chi_e$ — Fig. 21

Figure 20: $\chi_e$ in magnetically quiescent PPCD MST plasma; experimental data from Sarff et al. PPCF 45, A457 (2003).

Figure 21: SSPX $\chi_e$ on axis decreases as $T_e$ increases; expt. data from McLean et al., PoP 13, 056105 (2006).
SUMMARY AND CONCLUSIONS

- **Tests** — Paleoclassical model of radial electron heat transport has been compared with experimental data from various radial regions of a wide variety of axisymmetric resistive, current-carrying toroidal plasmas.

- **Regimes** — Paleoclassical transport found to be relevant in many regimes:
  
  - Core: irreducible minimum $\chi_e$ may be set by $\chi_e^{pc}$ when $T_e \lesssim T_e^{crit} \sim \frac{B^2}{3\bar{a}}\frac{1}{2} \text{keV}$
  - eITBs: reduced $\chi_e^{pc}$ at low order rationals reproduces RTP ECH “stair-step” data
  - eITB: for reversed shear with low turb-induced transport, $\chi_e^{pc}$ small but can dominate
  - Pedestal: paleoclassical transport may be dominant for $T_e$ in H-mode edge pedestals
  - Other conf concepts: Minimum $\chi_e$ may be set by $\chi_e^{pc}$ in spherical toks., RFPs, spheromaks

- **Conclusions** — $\chi_e^{pc}$ may provide the minimum $\chi_e$, unless it is exceeded by
  
  - Core, eITBs: ITG/TEM turbulent transport for $T_e \gtrsim T_e^{crit} \sim \frac{B^2}{3\bar{a}}\frac{1}{2} \text{keV}$
  - Core: sawtooth effects in the core ($\rho \lesssim 0.3$) of tokamak plasmas
  - Edge: resistive ballooning modes for $T_e \lesssim 300 \text{ eV}$ in L-mode edge plasmas
  - Core: magnetic-flutter-induced transport (RR) when magnetic fluctuations are large

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Annotated Bibliography

• Basic paleoclassical model in a sheared-slab magnetic field geometry:

• Axisymmetric magnetic field geometry, full kinetic analysis:

• Paleoclassical model, plus initial experimental comparisons and 19 possible tests:

• Derivation of key hypothesis of the paleoclassical model:
  “Key hypothesis of paleoclassical model,” UW-CPTC 06-3, Sept. 2006 (sub. to PoP).

• These recent, more detailed experimental comparisons to paleoclassical model:

• Preprints and reprints of these papers are available from
  http://homepages.cae.wisc.edu/~callen and http://www.cptc.wisc.edu