Surfaces: by definition

• An area bounded by an identifiable perimeter.
• In Computer Graphics, a 2D locus of points in 3D space, defined by mathematical methods.

Surfaces

• Surfaces may be:
  – Planar
  – Cylindrical/conic
  – Sculptured or freeform in shape

Mathematical methods for surface definition

• developed during and after WWII to aid the aircraft and shipbuilding industries.
• developments used to assist in the machining of complex 3-dimensional surfaces.

Parametric definition

• A means of accurately describing a surface in all three dimensional directions.
• These techniques have been adapted to computer-aided modeling to develop modelers capable of defining surface features

Recall the parametric line representation

Parametric representation of a line. The parameter \( u \), is varied from 0 to 1 to define all points along the line.

\[
X = X(u) \quad Y = Y(u)
\]
Expanding the 2D parametric technique we used for a line to define a surface, two parameters ($u$ and $v$) are used.

Points along edge $P_1P_2$ have the form of $P(u,0)$, along $P_3P_4$, $P(u,1)$ and so on.

By varying value of $u$ and $v$, any point on the surface or the edge of the face may be defined.

Curved Surface Patches

- Planar surface on previous slides defined by first degree curves (first power functions of $u$ and $v$)
- For more complex surface shapes, higher order curves are used.
- For example, a third order curve would use equations of the powers $u^1$, $u^2$, and $u^3$.

Surface Normal Vector

- A surface is defined by parametric equations in $u$ and $v$.
- Therefore at any given point there exist two tangent vectors (in $u$ and $v$ directions)
Surface Normal Vector

- The two tangent vectors define a tangent plane.
- The vector perpendicular to this plane is the normal vector of the surface at that point.
- It is determined by computing the cross product of the normal vectors.
- A planar surface will have a single normal vector direction.

Surface Normal at Given Point

Surface Curvature

- Taking the derivative of the two tangent vectors at any given point yields values of local curvature.
- Surface curvature is a measure of the rate-of-change of the curve slope. (how fast is the curve changing)

Surface Curvature

- Surface curvature is an important aspect in design.
  - Provides an indication of smoothness (local creases and indentations)
- Can be used to help in manufacturing.
  - Cutting tool size can be a function of maximum/minimum curvature

Gaussian Curvature

- At a point P on a surface, there will exist a Maximum and a Minimum curvature
  - (not necessarily in u and v directions)
- They are referred to as the Principal Curvatures
- The product of the Principal Curvatures is the Gaussian Curvature

For this surface...
Gaussian Curvature

- Gaussian curvature can be a useful tool for finding surface inflections.
- It indicates changes in surface direction.
  - For example, You can immediately spot a dent in the side of your car’s door because it is an unwanted inflection in the surface, even if the surface is still tangent and curvature continuous.

Gaussian Curvature

- Since Gaussian curvature is the product of the principle curvatures:
  - For convex or concave surface, it has a positive value
  - For planar surfaces it has a value of zero
  - And, it has a negative value at a saddle point
    - One curvature is positive, the other negative

Saddle Point

Saddle Point
Surface definition

- Ability to query database for surface information means:
  - possible to determine if a point lies off, on or on the boundary of a surface
  - often referred to as a "membership test"
- data may be used for functions such as
  - rendering, manufacturing, surface area determination

Rendering

- Results of membership test allow for the calculation of intersections between line of sight vectors and surface geometry.
- This allows for the "depth sorting" of features
  - This is, which surfaces (or parts of surfaces) are visible from a particular line of sight

Faceted models

- Some modelers limit model database or display representations to 1st order (planar) surface definitions
- Surfaces of the model are represented by one or more planar polygons
- Such models are known as polyhedral, polygonal or faceted models

Faceted models

- Some modelers may store multiple databases (exact and approximate) and use faceted format for display only (to simplify display)
- Display may be "cleaned-up" through rendering techniques.
  - Note that display alone does not indicate database storage

Faceted display and “smoothed” image

Creating Surfaces

- Complex shape generation facilitated through various methods
- These methods permit complex surface generation through the use of defining parent geometry.
- Parent geometry may be curves, vectors or specification data
Associativity

- A dependency that exists between model entities
- "child" entities are dependent upon their respective "parent geometry"
- For surface construction:
  - Parent geometry required for surface creation
  - Surface cannot exist without parent geometry (cannot be deleted if associative)

Categorizations of complex surfaces:

- surface of revolution
- tabulated cylinder
- ruled surface
- general sweep
- sculptured (Coons’ Patch)

Surface of Revolution

- Requires:
  - a shape curve (must be continuous)
  - a specified angle
  - an axis defined in 3D modelspace.
- Positive rotation direction usually based upon direction of axis vector.

Mathematic Definition of Surface of Revolution

- Any point on surface is a function of two parameters, \( t \) and \( \theta \)
  - \( T \) describes the shape to be rotated
  - \( \theta \) represents the angle of rotation

Tabulated cylinder

- Defined by projecting a shape curve (or profile) along a direction vector.
- Curvature in one direction only (along shape curve), linear in other direction.
Tabulated surface

- Linear interpolation between two bounding geometric elements (curves).
- Elemental division the same for each curve.
- Bounding curves must both be either geometrically open (line, arc) or closed (circle, ellipse).
- Curvature in one direction only.

Ruled surface

- Shape curve (or profile) is swept along a path defined by an arbitrary curve.
- Compare with:
  - Tabulated (path a vector)
  - Surface of revolution (path a single curve)

General sweep

- Shape curve and path for General Sweep.

Sculptured surface (Coons’ patch)

- Sometimes referred to as a “curve-mesh” surface.
- Most general of surface mesh definitions
- Generated by interpolation across a set of defining shape curves
Some sculpted surfaces generated in a technique called **lofting** or **blending**.

- A set of cross-sections curves are established.
- The system will interpolate the cross-sections to define a smooth surface geometry.

**Surface Model Construction**

- Individual surfaces must be: “intersected”, “trimmed”, “stitched” to one another to form a final shape
- May be a lack of connectivity between individual surfaces, e.g. tiny gaps, and overlaps between surfaces

**Surface models**

- A significant improvement over wireframes
  - Provide both visual and mathematical descriptions of model geometry.
- More computationally expensive than wireframe models.
  - Higher storage requirements, longer model regeneration time

**Surface Model Database**

- Permits direct assessment of certain model geometry such as:
  - Curve slope and curvature
  - Orientation of surface normal vectors
  - Surface curvature
  - Perimeter determination and surface area
Surface information:
• allows points on the area of a surface to be mapped mathematically.
  – valuable in the generation of cutting tool paths for computer numerically controlled (CNC) machining operations.
• used for the generation of visually realistic images through techniques such as
  – rendering
  – hidden edge removal.

Limitations of surface modeling techniques:
• Does not represent internal features of the model, no sense of volume.
• Models of limited value for volumetric and mass property analysis.
  – limited to certain shapes and features

Limitations of surface modeling techniques:
• Possible to construct surfaces which intersect upon themselves
• Multiple surfaces are needed to represent “real world” geometries.
  – Modeling software may not provide for adequate connectivity between these adjacent surfaces.

Limitations of surface modeling techniques:
• Connectivity
  – possibility exists for discontinuities at the intersections of adjacent surfaces.
  – no guarantee of continuity between adjacent surfaces
• Model representation in this case is simply a collection of surfaces with no connectivity (topological) information.