\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

[1]

\[ \rho \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} \]  

[2]

\[ \rho \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = \frac{\partial p}{\partial y} \]  

[3]

N-S includes mass (?)

3 coupled diff eqs.

want to solve for

\[ u(x,y) \]
\[ v(x,y) \]
\[ p(x,y) \]

for all but simplest geometries, we must resort to CFD; we represent \( u, v, p \) with discrete variables @ fixed locations (Eulerian description)

Lagrangian possible but Eulerian easier
STEP 1
Geometry Combit (looks like CAD)

- easy for simple shapes, but not always
  (internal flow under hood of car, using CAD drawings of everything inside)

Notation

\[ L_X = (m-1) \Delta X, \quad \Delta X = \frac{L_X}{m-1} \]
\[ L_Y = (n-1) \Delta Y \]
\[ \Delta X = \Delta Y \text{ for square} \]

Step 2 - apply differential analysis at the nodes
convert our differential equations to algebraic difference equations
we know \( f'(x) \) and \( f''(x) \)

we know \( f'(a) \) and \( f'(x) \)

use Taylor series

\[ f(x) = f(a) + f'(x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \ldots \]

for our setup

\[ u_{i+1, j} = u_{ij} + \frac{\partial u}{\partial x} \bigg|_{ij} \Delta x + \frac{\partial^2 u}{\partial x^2} \bigg|_{ij} \frac{\Delta x^2}{2!} + h.o.t. \]

kind of ignore \( j \)

\[ \frac{\partial u}{\partial x} \bigg|_{ij} = \frac{u_{i+1,j} - u_{ij}}{\Delta x} \quad - \frac{\partial^2 u}{\partial x^2} \frac{\Delta x}{2!} + h.o.t. \]

Call this a first-order method \( O(\Delta x) \) truncation error \( TE \)

Can already see small mesh is nice

Fluent has options which are higher order

use similar method for other \( 5 \) derivatives:

\[ \rho \left[ u_{ij} \frac{u_{i+1,j} - u_{ij}}{\Delta x} + v_{ij} \frac{(u_{ij} + v_i - u_{i+1,j} - v_{ij})}{\Delta y} \right] = -P_{i+1,j} - P_{ij} \]
Step 3: Init and boundary conditions (IC & BC)

Upwards

guess for steady flows.
real for unsteady flows.

Step 4: Solve numerical scheme

By marching, explicit 1st order 1-D wave equation

\[ \frac{\partial u}{\partial t} + C \frac{\partial u}{\partial x} = 0 \]

aka

\[ \frac{\partial u}{\partial t} + C \frac{\partial u}{\partial x} = 0 \]

\[ u^n_{i+1} = u^n_i - C \frac{\Delta t}{\Delta x} (u^n_i - u^n_{i-1}) \]

We know all these at current step n from initial conditions

\[ \frac{u^n_i - u^n_{i+1}}{\Delta t} + C \frac{u^n_i - u^n_{i-1}}{\Delta x} = 0 \quad C > 0 \]

\[ u^{n+1} = u^n_i - C \frac{\Delta t}{\Delta x} (u^n_i - u^n_{i-1}) \]

Residual = 0.001 = R

\[ R = \sum | \text{rate of mass creation} | \]

\[ \sum |m^{n+1} - m^n| \]

Q: Many small volume analysis vs. this differential approach; finite volume scheme vs. finite diff. scheme.