\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

1. Conservation of mass

\[ P \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} \]  

2. Conservation of momentum

\[ P \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial P}{\partial y} \]  

3. Conservation of momentum

\[ \begin{align*} 
\text{Navier–Stokes} & \\
\text{Conservation of momentum} & \\
\text{assume} & \\
\rightarrow & 2-D \text{ no } z/w \\
\rightarrow & \text{ incompressible } \frac{\partial u}{\partial x} = 0 \\
\rightarrow & \text{ inviscid } \mu = 0 \\
\rightarrow & \text{ steady } \text{ no } \frac{\partial u}{\partial t} \text{ etc.} \\
\rightarrow & \text{ laminar } (\text{no turbulence}) 
\end{align*} \]
3 coupled equations
\[ \Rightarrow \] Solve for 3 things
\[ p(x, y), u(x, y), V(x, y) \]

\[ V = k \rho s - k x_j \]

no analytical solution except for very simple geometries

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**STEP 1**
Geometry (Gambit)

\[ u, \theta, V, p \] @ each node

---

**STEP 2**
apply differential analysis to nodes
convert diff. eqs. to algebraic difference eqs.

Taylor Series
\[ f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \text{h.o.t.} \]

\[ \text{h.o.t.} \] 2nd
**STEP 3**

Initial and boundary conditions: IC + BC

```
BC: flow normal to walls = 0
BC: inlet<=>P  u, v  outlet<=>P
IC: guess (u=1 m/s) or starting value for steady flows
    real IC for unsteady flow
```

**STEP 4**

Solve numerical scheme (FLUENT)

- Continuity equation:
  \[
  \frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta y} = 0
  \]

- X-momentum:
  \[
  P \left[ u \frac{\Delta u}{\Delta x} + v \frac{\Delta u}{\Delta y} \right] = -\frac{\Delta p}{\Delta x}
  \]

- Y-momentum:
  \[
  P \left[ v \frac{\Delta v}{\Delta x} + v \frac{\Delta v}{\Delta y} \right] = -\frac{\Delta p}{\Delta y}
  \]

Residual = difference between idetically satisfying these equations